The SOM Numbers (part I)

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Abstract: This study delves into the interconnection between square and oblong numbers and their relation to number theory, significantly enhancing the understanding of numerical structure and prime number distribution. Initially, we examine the interweaving pattern between square and oblong numbers, revealing a periodic classification of integers and establishing an analogy with the interlocked teeth of a zipper. This analysis lays the foundation for an in-depth revision of Legendre's Conjecture, offering new insights into the distribution of prime numbers along the numerical line and uncovering previously unexplored mathematical connections.
We advance in the field of quadratic sequences by introducing what we term the "3 Basic Theorems of Quadratics." These theorems are fundamental for understanding the properties and behaviors of quadratic functions, including the application of Taylor shift and offset.
We develop a comprehensive atlas mapping all sequences of the type "quadratic minus an element of the quadratic," providing extensive documentation and categorization of these sequences. This atlas allows for the exploration of a wide range of behaviors and patterns in quadratic functions. Furthermore, we demonstrate how finite sequences, specifically trios, can uniquely characterize each of the infinite quadratic sequences.
Complementing the atlas, we create a universe of matrices of quadratic sequences, logically and structurally organizing all sequences. These matrices illuminate patterns, symmetries, and recurring properties in quadratic sequences, offering a powerful tool for solving open mathematical problems. The ultimate goal is to establish a definitive system for organizing all connections between quadratic sequences, fostering significant advancements in various areas of mathematics, such as number theory, algebra, and their practical applications.
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1 Introduction

*In a school, a teacher was hired to provide private lessons to 50 students. To better understand each one of them, he decided to have lunch individually with each student at a local restaurant, spending R$50.00 per meal. Each lunch outing with a student cost 2*R$50.00. With 50 students, the total cost equated to the value of two squares of 50.*

Reflecting on the expenses, the teacher concluded that a collective lunch with all 50 students at once would allow interaction with each of them. Recalculating the costs for this collective lunch, the total came to $50*(50+1) = 50^2+50$, resulting in an oblong number cost. This bothered the teacher, as it involved paying for a square of 50 plus another 50 personally. Nearly halving the first cost, the only remaining step was to discount his own meal to achieve a comfortable balance.

*Subsequently, the teacher negotiated with the restaurant management for a free meal. The manager, realizing the ease of calculating 50*50 compared to 51*50, considered a 2% discount for 1 in 51 and agreed to the negotiation. The teacher's costs were reduced to that of only one square of 50.*

In the first lesson to the students, the teacher emphasized the non-negotiability of mathematics, stating that the relationship of 1 to all integers remains intact. While humans find symmetry in dividing by 2, nature always presents an 'oblong term (n±1)' in a slightly asymmetrical proportion.

Furthermore, the teacher explained that subtracting 1 from a perfect square yields a product, whereas doing so from an oblong number reveals a unique beauty within polynomial prime sequences.

This study commences with a detailed analysis of the interconnection between square and oblong numbers, akin to the interlocking teeth of a zipper. This mechanism underpins a comprehensive periodic classification of integers, demonstrating how categories of square numbers, oblong numbers, and squares minus one encompass and structure the entire set of integers. We advance upon Legendre's conjecture, delving into its extensive implications in the distribution of prime numbers across the numerical spectrum. This segment aims to unveil previously unexplored mathematical relations and connections, offering a fresh perspective on prime number distribution.

We expand the study to include not only the Theorem of the sum of odd numbers but also the incorporation of even numbers and the entirety of positive integers. In this pursuit, we establish what we term the 3 Basic Theorems of Quadratics, laying the foundation for understanding the properties and behaviors of quadratic functions. We deliberate on specific quadratic sequences, such as (square minus square), (oblong minus oblong), and (triangular minus triangular), utilizing Taylor displacement and offset techniques to uncover hidden patterns within these structures. A notable point of interest is the correlation between Taylor shift and the inherent (1/integer) slope in quadratic sequences in the XY plane.

From these basic theorems, the study progresses to generalize and map all sequences of the type "quadratic sequence minus an element of the quadratic." This atlas is an extensive collection that documents and categorizes each sequence formed by subtracting a quadratic function element from the function itself. This approach allows for the exploration and understanding of the myriad behaviors and patterns of quadratic functions. Once all quadratic CGs are mapped, any quadratic sequence of prime numbers can be obtained as (CG + constant). We demonstrate, practically in quadratics, how a finite sequence of elements, in this case, trios, can be uniquely used to characterize each of the infinite polynomial sequences.

Building on the atlas, the study develops a "universe of matrices of quadratic sequences." Here, "matrix" is understood as a systematic arrangement or framework that organizes all sequences of quadratic sequences in a logical and structured manner. These matrices assist in visualizing and comprehending the relationships and interconnections among different quadratic sequences. They illuminate patterns, symmetries, and recurring properties that can be leveraged to solve open mathematical problems.

The ultimate goal of this study is to create a system that definitively organizes all the linkages between quadratic sequences. We categorize and elucidate all possible relations and interactions among these sequences. Such an organization is fundamental for advancements in various mathematical areas, including number theory, algebra, and potentially in practical applications, such as in physics or economics, where quadratic models are frequently utilized.

The natural evolution of this work leads to the presentation of this universe of matrices in three dimensions, which will be the focus of a forthcoming study. This progression into a 3D matrix universe promises to deepen our understanding of these complex relationships and offer new insights, potentially unlocking further mathematical mysteries and applications. This forthcoming study will aim to expand our framework and explore the spatial properties and dynamics of these quadratic sequences in a three-dimensional context.

2 The Interleave between Squares and Oblongs

In this item, we delve into the fascinating relationship between square and oblong numbers, focusing on their successors and predecessors. This exploration reveals intriguing patterns similar to the well-known alternation between even and odd numbers.

2.1 Successor Square

For any integer y, it´s square is y^2. The successor square, denoted as $(y+1)^2$, exhibits a relationship with oblong numbers:

$$(y+1)^2 = y^2 + 2y + 1 = y^2 + y + (y+1) = y(y+1) + (y+1) = (y+1)(y+1)$$

This formulation reveals that the successor square is the sum of the oblong of y and the successor of y:

$$square[y+1] = (oblong[y]) + (successor\ of\ y)$$

For instance, if $y = 5$, it´s square is $y^2 = 25$, and the successor square is calculated as:

$$(5+1)^2 = 5^2 + 2 * 5 + 1 = 5^2 + 5 + (5+1) = 5(5+1) + (5+1) = (5+1)(5+1)$$

In the midpoint between the square and its successor square lies an oblong number:

$$\frac{y^2 + (y+1)^2 - 1}{2} = \frac{y^2 + y^2 + 2y + 1 - 1}{2} = \frac{2y^2 + 2y}{2} = y^2 + y = y(y+1)$$
$$= oblong\ number$$

2.2 Predecessor Square

Similarly, for any integer y with its square as y^2, the predecessor square, denoted as $(y-1)^2$, manifests a distinctive relationship with oblong numbers:

$$(y-1)^2 = y^2 - 2y + 1 = y^2 - y - (y-1) = y(y-1) - (y-1) = (y-1)(y-1)$$

The predecessor square is, thus, expressed as the oblong of $-y$ minus the predecessor of y:

$$square[y-1] = (oblong[-y]) - (predecessor\ of\ y)$$

For example, if y = 5, its square $y^2 = 25$, and the predecessor square is computed as:

$$(5-1)^2 = 5^2 - 2 * 5 + 1 = 5^2 - 5 - (5-1) = 5(5-1) - (5-1) = (5-1)(5-1)$$

In the middle of the square and its predecessor square resides an oblong number:

$$\frac{y^2 + (y-1)^2 - 1}{2} = \frac{y^2 + y^2 - 2y + 1 - 1}{2} = \frac{2y^2 - 2y}{2} = y^2 - y = y(y-1)$$
$$= oblong\ number$$

2.3 Successor Oblong

Moving to oblongs, where the oblong of y is given by $(y^2 + y)$, the successor oblong for any integer y is expressed as:

$$(y+1)^2 + (y+1) = square\big[successor[y]\big] + successor[y]$$

For instance, if $y = 5$, it´s oblong is $(y^2 + y) = 30$, and the successor oblong is determined as:

$$(5+1)^2 + (5+1) = square\big[successor[5]\big] + successor[5] = square[6] + 6$$

In the middle of the oblong and its successor oblong lies a square number:

$$\frac{(y^2 + y) + \big((y+1)^2 + (y+1)\big)}{2} = \frac{y^2 + y + y^2 + 2y + 1 + y + 1}{2} = \frac{2y^2 + 4y + 2}{2} =$$
$$= y^2 + 2y + 1 = (y+1)^2 = square\ number$$

$$\frac{(y^2+y)+\left((y-1)^2+(y-1)\right)}{2}=\frac{y^2+y+y^2-2y+1+y-1}{2}=\frac{2y^2}{2}=y^2$$
$$=square\ number$$

2.4 Predecessor Oblong

Lastly, for any integer y, with its oblong as (y^2-y), the predecessor oblong is expressed as:

$$(y-1)^2-(y-1)=square\left[predecessor[y]\right]-predecessor[y]$$

For instance, if $y=5$, it´s oblong is $(y^2-y)=20$, and the predecessor oblong is calculated as:

$$(5-1)^2-(5-1)=square\left[predecessor[5]\right]-predecessor[5]=square[4]-4$$

In the midpoint between the oblong and its predecessor oblong, we find a square number:

$$\frac{(y^2-y)+\left((y-1)^2-(y-1)\right)}{2}=\frac{y^2-y+y^2-2y+1-y+1}{2}=\frac{2y^2-4y+2}{2}$$
$$=y^2-2y+1=(y-1)^2=square\ number$$

$$\frac{(y^2-y)+\left((y+1)^2-(y+1)\right)}{2}=\frac{y^2-y+y^2+2y+1-y-1}{2}=\frac{2y^2}{2}=y^2$$
$$=square\ number$$

2.5 Summary

Whereas all the (square – 1) numbers precede a square number, the distribution for square and oblong numbers follows:

$$\frac{Square[y]+Square[y\pm 1]-1}{2}=\frac{y^2+(y\pm 1)^2-1}{2}=y(y\pm 1)=oblong\ number$$

$$\frac{Oblong[y]+Oblong[y\pm 1]}{2}=\frac{(y^2\mp y)+\left((y\mp 1)^2\mp(y\mp 1)\right)}{2}=square\ number$$

So

- Between consecutive square numbers, there is exactly one oblong number.
- Between consecutive oblong numbers, there is exactly one square number.
- Each square number, has both a predecessor and a successor oblong number $y^2\pm y$.
- Each oblong number, has both a predecessor and a successor square number $(y-1)^2$ and y^2, or y^2 and $(y+1)^2$.

2.6 Conclusions

This study illustrates a rhythmic alternation between square and oblong numbers, akin to the alternation between even and odd numbers, but with a distinct quadratic nature. This difference allows for other numerical types to be present in the intervals between squares and oblongs, with the gaps following a parabolic curve.

An interesting observation is that square and oblong numbers consist of a finite number of primes and an infinite number of composites. Therefore, primes not classified as square and oblong distribute themselves in a repetitive pattern between them, a key factor in approaching Legendre's Conjecture.

Finally, the sequence found at OEIS https://oeis.org/A002620 displays the interleaving pattern of squares and oblongs, shedding light on their intricate relationship.

3 The SOM numbers and Their Relationship with Primes

In the realm of mathematical sequences, the SOM numbers, designated as A006446 in the Online Encyclopedia of Integer Sequences (OEIS), embody a fascinating interplay among square numbers (S), oblong numbers (O), and (square – 1) numbers (M). This study delves into the intricate relationships and patterns formed by these numbers.

Whereas all the (square – 1) numbers precede a square number, the distribution for square and oblong numbers follows:

- Between consecutive (square – 1) numbers, lies an alternating pair of one oblong number and one square number.
- Between consecutive square numbers, lies an alternating pair of one oblong number and one (square – 1) number.
- Similarly, within the sequence of consecutive oblong numbers, an alternating pair emerges—one (square – 1) number and one square number.

The elegance of these interleaved sequences is encapsulated in the SOM numbers, a term coined in this study to represent the interwoven trio of square numbers (S), oblong numbers (O), and (square – 1) numbers (M).

3.1 Understanding the SOM Numbers

Square Numbers (S): These are numbers formed by the square of an integer. Except for the number 1, all square numbers are composite.

Oblong Numbers (O): Formed by the product of two consecutive integers, all oblong numbers are composite, except for number 2.

(Square – 1) Numbers (M): These are numbers that are one less than a square number. All positive (square-1) numbers are composite, except for number 3.

3.2 The Kircheri Triangle and SOM Numbers

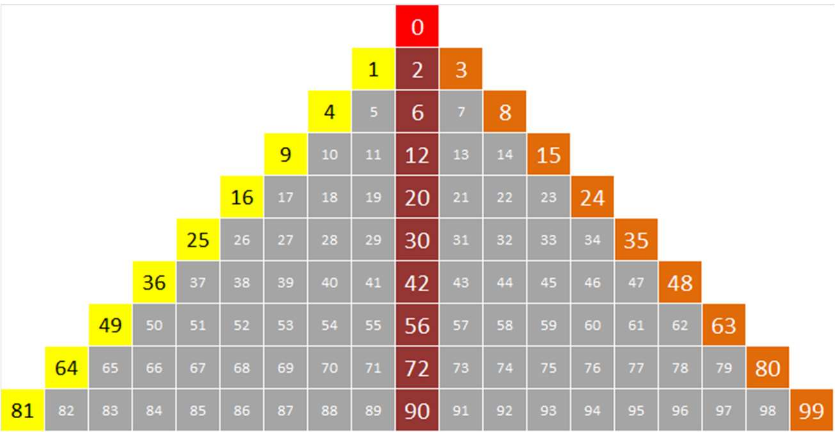


Figure 1. C001202 The SOM numbers https://oeis.org/A006446.

The SOM numbers occupy the side edges and the central axis of the Kircheri triangle (Kircheri, P. Athanasii. (1664) Mundus Subterraneus, pg. 24).

All odd prime numbers greater than 3 are located between a square number and its subsequent oblong number, or between an oblong number and the subsequent (square – 1) number. This is a direct consequence of the fact that, except for the initial cases, square, oblong, and (square – 1) numbers are composite.

Please consult *C000885 The Athanasii Kircheri triangle filled with non-negative integers* at https://www.mersenneforum.org/showthread.php?t=28110. The sum row by row is the product of two consecutive numbers multiplied by the sum of the two consecutive numbers:

$$\pm 2y\text{^}3 - 3y\text{^}2 \pm y = y(y-1)(2y-1) = oblong * odd = y(y-1)(y+(y-1))$$

Equivalent to https://oeis.org/A055112.

4 The Distribution of Primes Based on the Study 'Integers are only Primes and Composites'

This section explores the distribution of prime numbers in relation to square, oblong, and (square-1) numbers, in the context of the study "Integers are only Primes and Composites" [17], where the number 1 is considered the first positive odd prime number.

4.1 Prime Number Distribution

Under this framework, the distribution of prime numbers between a square number and its subsequent oblong number, or between an oblong number and its subsequent square number, pertains exclusively to odd primes. Notably:

SO Prime Numbers: Listed in https://oeis.org/A307508, these odd primes are found between a square number and its following oblong number. For example, {1, 5, 11, 17, 19, 29, 37, 41, ...}.

These are primes with a truncated square root equal to the rounded square root, as detailed in https://oeis.org/A063656.

OM Prime Numbers: Detailed in https://oeis.org/A334163, these odd primes are located between an oblong number and its following square number, such as {3, 7, 13, 23, 31, 43, ...}. These correspond to primes with a truncated square root not equal to the rounded square root, as found in https://oeis.org/A063657.

4.2 Takumi Sato's Classification of Prime Numbers

Applying Takumi Sato's classification [48] (Mersenne Forum link) to prime numbers, we can categorize them into four distinct groups:

Oblong Primes: Primes in https://oeis.org/A217575, "Numbers n such that floor(sqrt(n)) = floor(n/floor(sqrt(n)))-1." These include all oblong numbers in the sequence. The prime elements of this sequence are from https://oeis.org/A334163, replacing element 3 with 2. Example: {2, 7, 13, 23, 31, 43, 47, 59, 61, ...}.

Golden Primes: Primes in https://oeis.org/A217571, "a(n) = (2n(n+5) + (2n+1)(-1)^n - 1)/8." This includes https://oeis.org/A165900, "Values of the Fibonacci polynomial n^2 - n - 1." Prime numbers in this category follow https://oeis.org/A002327, "Primes of the form k^2 - k – 1," expanded to include 1 and -1. Example: {-1, 1, 5, 11, 19, 29, 41, 71, 89, ...}.

Square Primes: Primes in https://oeis.org/A217570, "Numbers n such that floor(sqrt(n)) = floor(n/(floor(sqrt(n))-1))-1." This sequence includes all square numbers greater than 4, with prime numbers being {17, 37, 53, 67, 83, ...}.

(Square-1) Primes: Primes in the (square – 1) sequence. This sequence is finite and includes {-1, 3}.

4.3 Expanding Legendre´s Conjecture

Considering that between two consecutive square numbers there is an oblong number, and between two consecutive oblong numbers there is a square number, it follows that all prime numbers must be situated between them. Prime numbers can only appear along the numerical line between squares and oblongs.

Given this interleaving of square and oblong numbers, we can extend Legendre's Conjecture:

- Between one square number and the next oblong number, there is always a prime number.
- Between an oblong number and the next square number, there is always a prime number.
- Between two square numbers, there are always at least two prime numbers.
- Between two oblong numbers, there are always at least two prime numbers.
- This suggests that Legendre's Conjecture could also be effectively stated in its equivalent form: *“There is always a prime number between two oblong numbers.”*

5 The Taylor Shift between Squares and Oblongs

In this section, we delve into the Taylor shift observed between the curves of square and oblong numbers, using calculus to understand their interrelationships.

5.1 Analyzing the Derivatives of Square and Oblong Curves

We begin by examining the derivatives of the square and oblong number curves:

Oblong Numbers Derivatives:

$$\frac{d}{dy}\left(\frac{y^2+(y-1)^2-1}{2}\right)=\frac{d}{dy}(y^2-y)=2y-1=2\left(y-\frac{1}{2}\right)$$

$$\frac{d}{dy}\left(\frac{y^2+(y+1)^2-1}{2}\right)=\frac{d}{dy}(y^2+y)=2y+1=2\left(y+\frac{1}{2}\right)$$

Square Numbers Derivatives:

$$\frac{d}{dy}\left(\frac{(y^2-y)+\left((y-1)^2-(y-1)\right)}{2}\right)=\frac{d}{dy}(y-1)^2=2y-2=2(y-1)$$

$$\frac{d}{dy}\left(\frac{(y^2-y)+\left((y+1)^2-(y+1)\right)}{2}\right)=\frac{d}{dy}(y)^2=2y=2(y-0)$$

$$\frac{d}{dy}\left(\frac{(y^2+y)+\left((y-1)^2+(y-1)\right)}{2}\right)=\frac{d}{dy}(y)^2=2y=2(y-0)$$

$$\frac{d}{dy}\left(\frac{(y^2+y)+\left((y+1)^2+(y+1)\right)}{2}\right)=\frac{d}{dy}(y+1)^2=2y+2=2(y+1)$$

5.2 Understanding the Taylor Shift

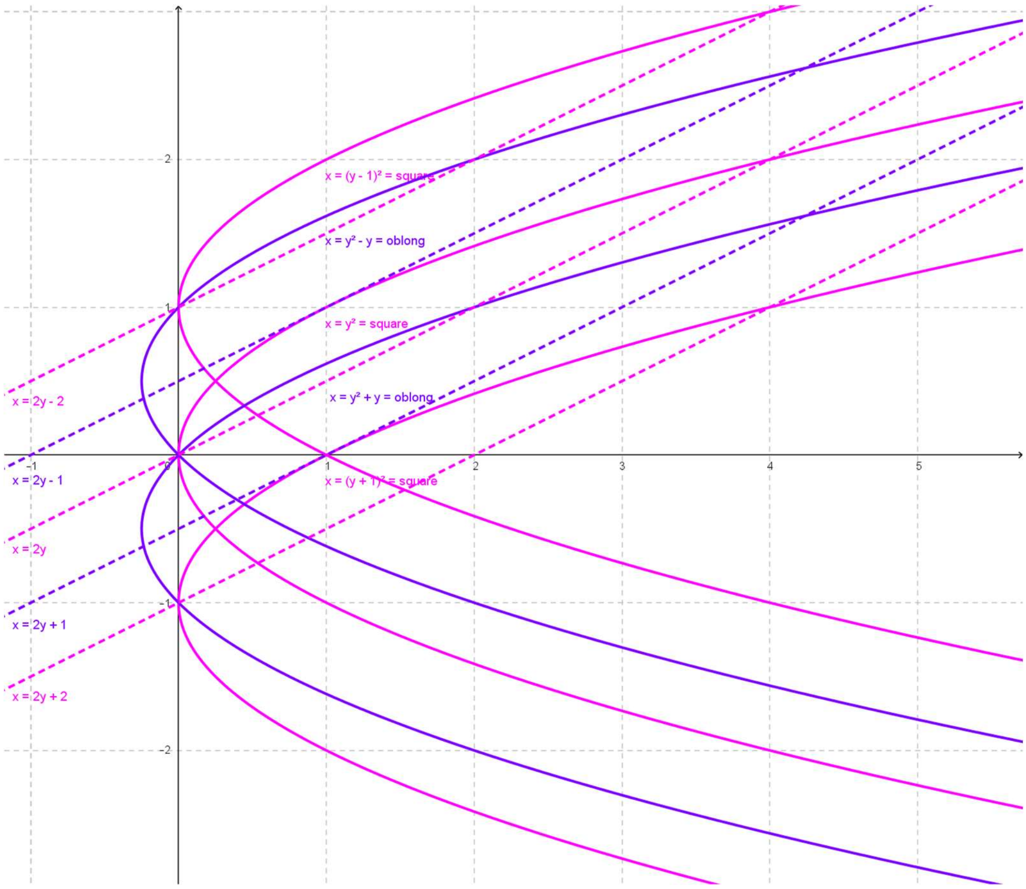


Figure 2. C001113 in the study visualizes these quadratic functions along with their linear derivative curves on the XY plane. This graphical representation aids in understanding the dynamic between the original functions and their derivatives.

The Taylor shift between the consecutive oblong and square curves is determined to be $1/2$ (0.5). This conclusion is drawn from observing the sequence of roots $\{-1, -0.5, 0, 0.5, 1\}$ of the derivatives of the consecutive oblong and square linear curves.

An alternative method to reach this conclusion is by following the Y-coordinate of the symmetry point in the above-mentioned curves.

5.3 Expanding to Squares and Oblongs Curves with Offset

In addition to the Taylor shift, we explore the offset in the XY plane for the squares and oblong numbers. This comprehensive analysis includes all Taylor shift-and-fit scenarios for these numbers:

SUB-type Parabolas: Representing squares or (squares sequence minus a square number), depicted in pink color.

DES-type Parabolas: Representing oblongs or (oblongs sequence minus an oblong number), shown in violet color.

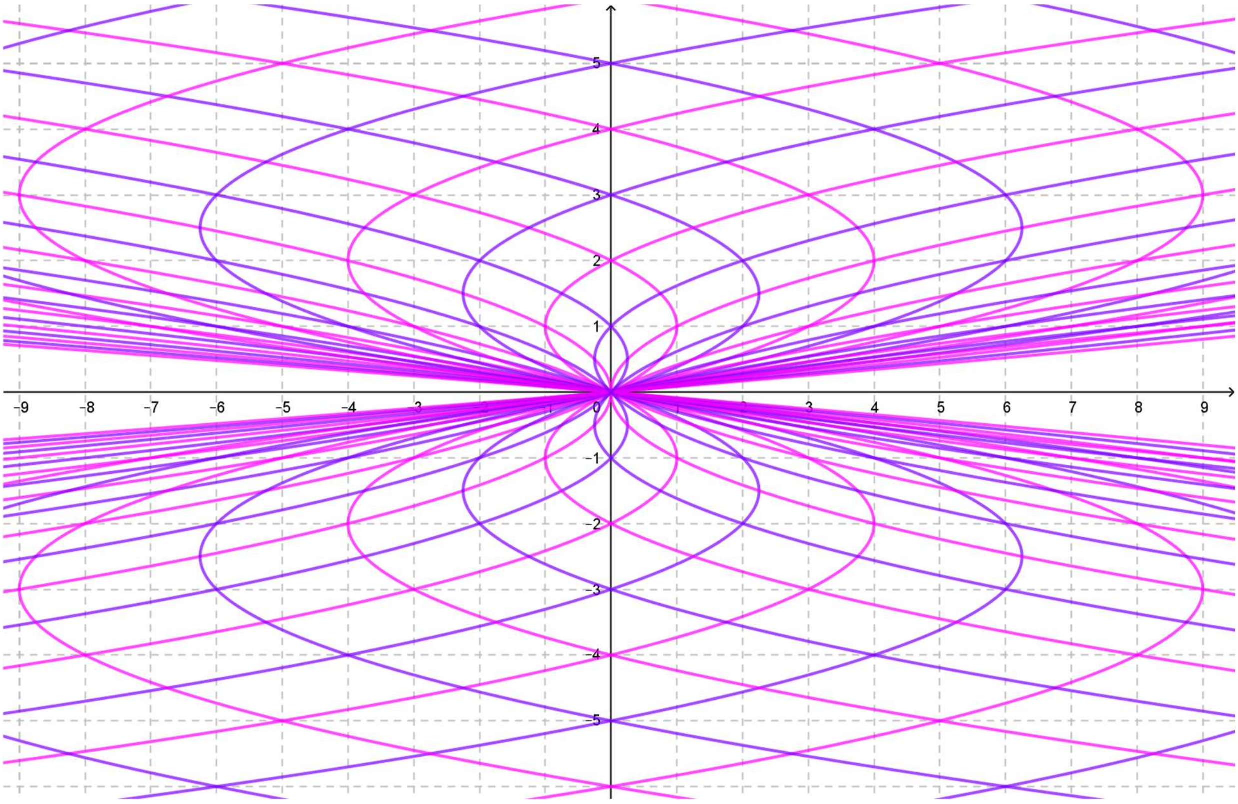


Figure 3. C001114 in the study provides a visual representation of the squares and oblong Taylor shift-and-fit in the XY plane.

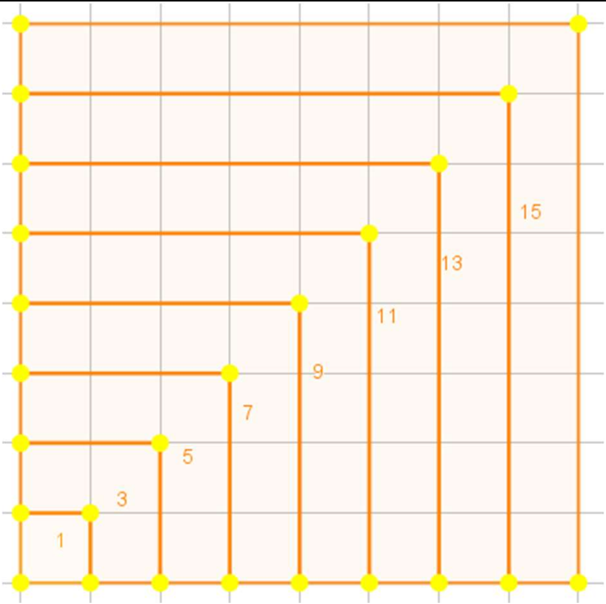
5.4 Conclusion

This exploration of the Taylor shift between square and oblong polynomial curves offers significant insights into their mathematical relationship. The use of derivatives and symmetry points, coupled with visual aids, allows for a deeper understanding of these intriguing mathematical phenomena.

For a more comprehensive analysis of this topic, the study "Exploring Parabolic Patterns in the Map of Integers and Divisors (MID) Framework"[14] provides an extensive exploration. This study delves deeper into the nuances of parabolic patterns and their implications within the MID framework, offering valuable perspectives that complement and expand upon the findings presented here.

6 Theorem of the Sum of Odd Numbers: A Visual Proof

The theorem that the sum of the first $'n'$ positive odd numbers is a square number is elegantly demonstrated through a visual sequence:



$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 5 = 9$$
$$1 + 3 + 5 + 7 = 16$$
$$1 + 3 + 5 + 7 + 9 = 25$$
$$1 + 3 + 5 + 7 + 9 + 11 = 36$$
$$\dots \textit{and so on.}$$

Figure 4. C000800, highlights how each sum results in a perfect square, forming the foundation of our exploration into the relationships between different number sequences

6.1 Partial Sum and offset

By applying the principle of partial sums to the sequence of odd numbers, we derive the sequence of square numbers. The formula is represented as:

$$y^2 = \sum_{1}^{y} (2y - 1)$$

This relationship holds true because we define the first odd positive element of all the sums as being 1. If we were to start the summation from the index $y = 2$, i.e., the partial sum of positive odd numbers starting with 3, then we would obtain the sequence of (square-1) numbers:

$$y^2 - 1 = \sum_{2}^{y} (2y - 1)$$

Similarly, if we start the summation from the index $y = 3$, or the partial sum of positive odd numbers beginning with 5, we will obtain the sequence of (square-4) numbers:

$$y^2 - 4 = \sum_{3}^{y} (2y - 1)$$

Generalizing this, for any starting index $(n+1)$, we have:

$$y^2 - n^2 = (y+n)(y-n) = \sum_{n+1}^{y} (2y-1)$$

Since the square number sequence is a SUB-type[18] sequence, all sequences in the form of $y^2 - n^2 = \sum_{n+1}^{y}(2y-1)$ will be SUB-type sequences. As these sequences are the result of partial sums of odd numbers, the resulting elements will always alternate between even and odd.

This generalization provides a clear understanding of the impact of varying the starting point in our summation. Specifically, this variation in the starting point can be interpreted as a change in the offset[18], corresponding to an integer-valued Taylor shift[18] within the sequence of odd numbers. By adjusting this offset, we effectively alter the final sequence, leading to a transition from perfect square numbers to their related forms in the (square sequence minus square number) pattern.

6.2 Partial Sum and Integral of Odd Numbers

When we interpret the partial sums of the sequence of odd numbers as integrals, we uncover interesting relationships. To align the integral of the odd number sequence with the square number sequence, we apply a Taylor shift along the Y-axis and a Eureka shift along the X-axis. This can be seen in the equation:

$$Y[y] = \int (2y-1)dy = y^2 - y + c = oblong + c$$

This means that the index shift value and constant c must be applied in such a way that the result is a square number y^2. This can be done only with the use of Taylor-shift-and-fit[18].

The Taylor shift must follow 5.2 above, $h = 0.5$, and the Eureka shift adjusts the constant $c = -(h^2 - h) = 0.25$ in the integral to fit in integer coordinates. Taylor shift acts in both directions Y-axis and X-axis. Eureka shift, only driven by constant c, acts just in the X-axis direction. Eureka shift acts to compensate for the X-axis shift produced by Taylor shift.

As an illustration, consider the following table that applies these shifts to various values of y:

Tally	y	$y^2 - y$	$c = -(h^2 - h)$	$y^2 - y + c$
1	0.5	-0.25	0.25	0
2	1.5	0.75	0.25	1
3	2.5	3.75	0.25	4
4	3.5	8.75	0.25	9
5	4.5	15.75	0.25	16
6	5.5	24.75	0.25	25
7	6.5	35.75	0.25	36
8	7.5	48.75	0.25	49
9	8.5	63.75	0.25	64
10	9.5	80.75	0.25	81
11	10.5	99.75	0.25	100

Figure 5. C000800 Integral of the odd numbers resulting in the square numbers.

Because we need apply a Taylor shift in the amount of $h = 0.5$, then, the index y start in 0.5.

When we apply a Taylor shift in amount of h in $Y[y] = y^2 - y$:

$Yº[y] = (y+h)^2 - (y+h) = (y^2 + 2hy + h^2) - y - h = y^2 + (2h-1)y + h^2 - h$

For the special case $h = \frac{1}{2}$, we have $(2h-1) = 0$. Then

$$Yº[y] = (y+h)^2 - (y+h) = y^2 + (h^2 - h)$$

$$Yº[y] = \left(y + \frac{1}{2}\right)^2 - \left(y + \frac{1}{2}\right) = y^2 - \frac{1}{4}$$

So, applying a Eureka shift for $c = -(h^2 - h) = +\frac{1}{4}$ then

$$Yº[y] = (y+h)^2 - (y+h) - (h^2 - h) = y^2$$

We can say, $Y[y] = y^2 - y + \frac{1}{4}$ with a Taylor shift $h = \frac{1}{2}$ is $Yº[y] = \left(y + \frac{1}{2}\right)^2 - \left(y + \frac{1}{2}\right) + \frac{1}{4} = y^2 = square$.

In integral form:

$$square = y^2 = \int_{0.5}^{\frac{2y+1}{2}} (2y-1)dy = \int_{0.5}^{\frac{next\ odd}{2}} (odd)dy = \int_{0.5}^{y+0.5} (2y-1)dy$$

6.3 Offset in Partial sum and Integral of the odds

By applying an offset to the partial sum of square numbers, we can observe distinct results. Applying an offset of one yield:

$$3 = 3$$
$$3 + 5 = 8$$
$$3 + 5 + 7 = 15$$
$$3 + 5 + 7 + 9 = 24$$
$$3 + 5 + 7 + 9 + 11 = 35$$
$$\ldots$$

These results correspond to the (square-1) numbers, as listed in https://oeis.org/A005563. The mathematical expression for this is:

$$square - 1 = y^2 - 1 = \int_{1.5}^{\frac{2y+1}{2}} (2y-1)dy = \int_{1.5}^{\frac{next\ odd}{2}} (odd)dy = \int_{1.5}^{y+0.5} (2y-1)dy$$

Applying an offset of two results in:

$$5 = 5$$
$$5 + 7 = 12$$
$$5 + 7 + 9 = 21$$
$$5 + 7 + 9 + 11 = 32$$
$$\ldots$$

These are the (square-4) numbers, corresponding to https://oeis.org/A028347. The formula is:

$$square - 4 = y^2 - 4 = \int_{2.5}^{\frac{2y+1}{2}} (2y-1)dy = \int_{2.5}^{\frac{next\ odd}{2}} (odd)dy = \int_{2.5}^{y+0.5} (2y-1)dy$$

For any starting index $(n + 0.5)$, the relationship can be generalized as:

$$y^2 - n^2 = (y+n)(y-n) = \int_{n+0.5}^{\frac{2y+1}{2}} (2y-1)dy = \int_{n+0.5}^{\frac{next\ odd}{2}} (odd)dy = \int_{n+0.5}^{y+0.5} (2y-1)dy$$

Because it is more common to work with definite integrals using integer limits, we can employ the Taylor shift by an amount of $h = 0.5$ in the odd function $Y[y] = 2y - 1$, and we observe the classical equality:

$$square - n^2 = \int_{n}^{y} (2y)dy$$

The same observations for partial sums apply to partial integrals that result in sequences in the form of (square sequence minus square number).

Figure 6, titled C000776 Paraboctys $PS[-x^2+1, -x^2, -x^2+1]$ provides a visual representation of the (square sequence minus square number).

X-axis -->		-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y-axis -->	15	0	29	56	81	104	125	144	161	176	189	200	209	216	221	224	225	224	221	216	209	200	189	176	161	144	125	104	81	56	29	0
	14	-29	0	27	52	75	96	115	132	147	160	171	180	187	192	195	196	195	192	187	180	171	160	147	132	115	96	75	52	27	0	-29
	13	-56	-27	0	25	48	69	88	105	120	133	144	153	160	165	168	169	168	165	160	153	144	133	120	105	88	69	48	25	0	-27	-56
	12	-81	-52	-25	0	23	44	63	80	95	108	119	128	135	140	143	144	143	140	135	128	119	108	95	80	63	44	23	0	-25	-52	-81
	11	-104	-75	-48	-23	0	21	40	57	72	85	96	105	112	117	120	121	120	117	112	105	96	85	72	57	40	21	0	-23	-48	-75	-104
	10	-125	-96	-69	-44	-21	0	19	36	51	64	75	84	91	96	99	100	99	96	91	84	75	64	51	36	19	0	-21	-44	-69	-96	-125
	9	-144	-115	-88	-63	-40	-19	0	17	32	45	56	65	72	77	80	81	80	77	72	65	56	45	32	17	0	-19	-40	-63	-88	-115	-144
	8	-161	-132	-105	-80	-57	-36	-17	0	15	28	39	48	55	60	63	64	63	60	55	48	39	28	15	0	-17	-36	-57	-80	-105	-132	-161
	7	-176	-147	-120	-95	-72	-51	-32	-15	0	13	24	33	40	45	48	49	48	45	40	33	24	13	0	-15	-32	-51	-72	-95	-120	-147	-176
	6	-189	-160	-133	-108	-85	-64	-45	-28	-13	0	11	20	27	32	35	36	35	32	27	20	11	0	-13	-28	-45	-64	-85	-108	-133	-160	-189
	5	-200	-171	-144	-119	-96	-75	-56	-39	-24	-11	0	9	16	21	24	25	24	21	16	9	0	-11	-24	-39	-56	-75	-96	-119	-144	-171	-200
	4	-209	-180	-153	-128	-105	-84	-65	-48	-33	-20	-9	0	7	12	15	16	15	12	7	0	-9	-20	-33	-48	-65	-84	-105	-128	-153	-180	-209
	3	-216	-187	-160	-135	-112	-91	-72	-55	-40	-27	-16	-7	0	5	8	9	8	5	0	-7	-16	-27	-40	-55	-72	-91	-112	-135	-160	-187	-216
	2	-221	-192	-165	-140	-117	-96	-77	-60	-45	-32	-21	-12	-5	0	3	4	3	0	-5	-12	-21	-32	-45	-60	-77	-96	-117	-140	-165	-192	-221
Y[1]	1	-224	-195	-168	-143	-120	-99	-80	-63	-48	-35	-24	-15	-8	-3	0	1	0	-3	-8	-15	-24	-35	-48	-63	-80	-99	-120	-143	-168	-195	-224
Y[0]	0	-225	-196	-169	-144	-121	-100	-81	-64	-49	-36	-25	-16	-9	-4	-1	0	-1	-4	-9	-16	-25	-36	-49	-64	-81	-100	-121	-144	-169	-196	-225
Y[-1]	-1	-224	-195	-168	-143	-120	-99	-80	-63	-48	-35	-24	-15	-8	-3	0	1	0	-3	-8	-15	-24	-35	-48	-63	-80	-99	-120	-143	-168	-195	-224
	-2	-221	-192	-165	-140	-117	-96	-77	-60	-45	-32	-21	-12	-5	0	3	4	3	0	-5	-12	-21	-32	-45	-60	-77	-96	-117	-140	-165	-192	-221
	-3	-216	-187	-160	-135	-112	-91	-72	-55	-40	-27	-16	-7	0	5	8	9	8	5	0	-7	-16	-27	-40	-55	-72	-91	-112	-135	-160	-187	-216
	-4	-209	-180	-153	-128	-105	-84	-65	-48	-33	-20	-9	0	7	12	15	16	15	12	7	0	-9	-20	-33	-48	-65	-84	-105	-128	-153	-180	-209
	-5	-200	-171	-144	-119	-96	-75	-56	-39	-24	-11	0	9	16	21	24	25	24	21	16	9	0	-11	-24	-39	-56	-75	-96	-119	-144	-171	-200
	-6	-189	-160	-133	-108	-85	-64	-45	-28	-13	0	11	20	27	32	35	36	35	32	27	20	11	0	-13	-28	-45	-64	-85	-108	-133	-160	-189
	-7	-176	-147	-120	-95	-72	-51	-32	-15	0	13	24	33	40	45	48	49	48	45	40	33	24	13	0	-15	-32	-51	-72	-95	-120	-147	-176
	-8	-161	-132	-105	-80	-57	-36	-17	0	15	28	39	48	55	60	63	64	63	60	55	48	39	28	15	0	-17	-36	-57	-80	-105	-132	-161
	-9	-144	-115	-88	-63	-40	-19	0	17	32	45	56	65	72	77	80	81	80	77	72	65	56	45	32	17	0	-19	-40	-63	-88	-115	-144
	-10	-125	-96	-69	-44	-21	0	19	36	51	64	75	84	91	96	99	100	99	96	91	84	75	64	51	36	19	0	-21	-44	-69	-96	-125
	-11	-104	-75	-48	-23	0	21	40	57	72	85	96	105	112	117	120	121	120	117	112	105	96	85	72	57	40	21	0	-23	-48	-75	-104
	-12	-81	-52	-25	0	23	44	63	80	95	108	119	128	135	140	143	144	143	140	135	128	119	108	95	80	63	44	23	0	-25	-52	-81
	-13	-56	-27	0	25	48	69	88	105	120	133	144	153	160	165	168	169	168	165	160	153	144	133	120	105	88	69	48	25	0	-27	-56
	-14	-29	0	27	52	75	96	115	132	147	160	171	180	187	192	195	196	195	192	187	180	171	160	147	132	115	96	75	52	27	0	-29
	-15	0	29	56	81	104	125	144	161	176	189	200	209	216	221	224	225	224	221	216	209	200	189	176	161	144	125	104	81	56	29	0

Figure 6. C000776 Paraboctys $PS[-x^2+1, -x^2, -x^2+1]$. The (square sequence minus square number) sequences.

The table below displays a range of these sequences with their corresponding OEIS entries:

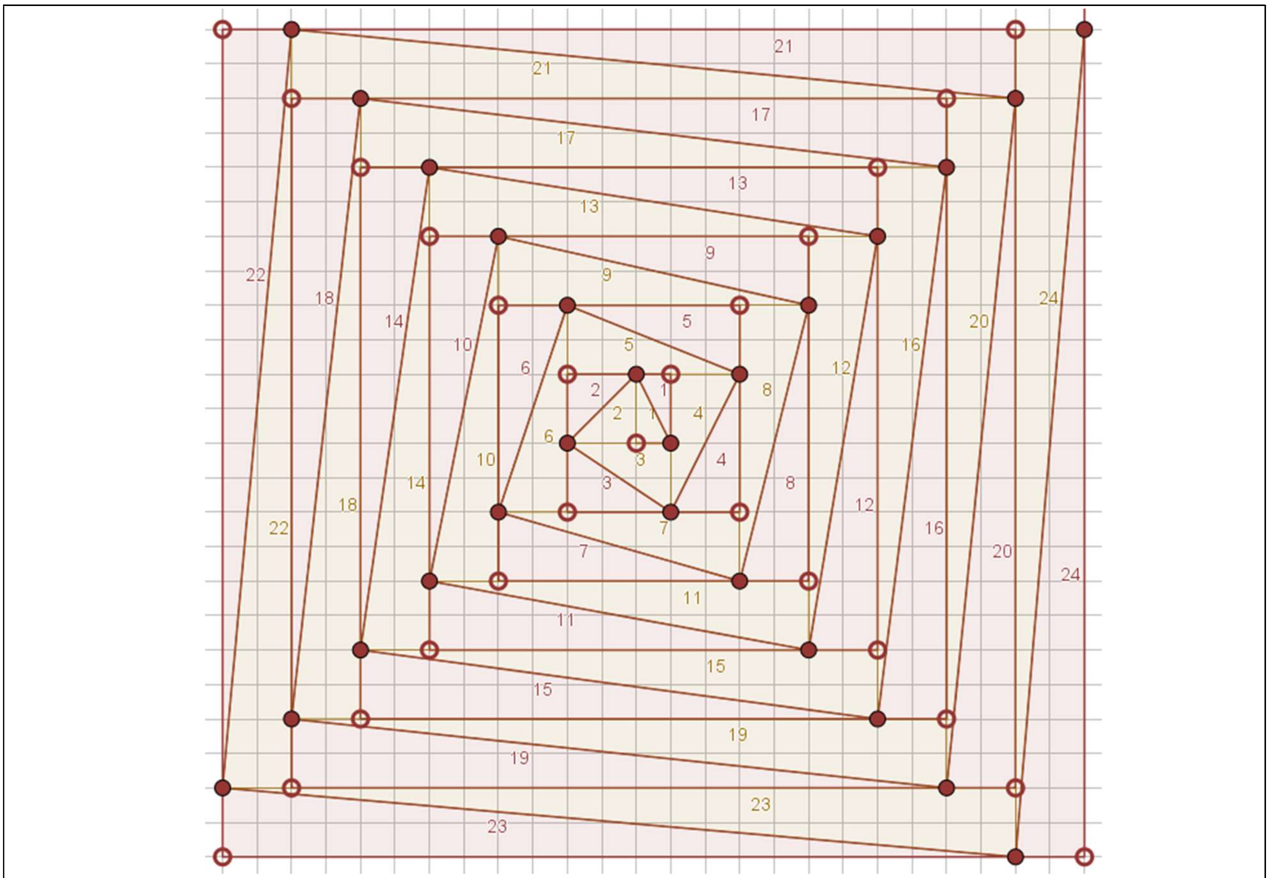
Horizontals	Verticals	SNYPO	Description	OEIS
X[x] = –x*(x ±0)	Y[y] = y*(y ±0)	C000800	Integers in the form of square–0.	http://oeis.org/A000290
X[x] = –x*(x ±2)	Y[y] = y*(y ±2)	C000801	Integers in the form of square–1.	https://oeis.org/A005563
X[x] = –x*(x ±4)	Y[y] = y*(y ±4)	C000802	Integers in the form of square–4.	https://oeis.org/A028347
X[x] = –x*(x ±6)	Y[y] = y*(y ±6)	C000803	Integers in the form of square–9.	https://oeis.org/A028560
X[x] = –x*(x ±8)	Y[y] = y*(y ±8)	C000804	Integers in the form of square–16.	https://oeis.org/A028566

X[x] = −x*(x ±10)	Y[y] = y*(y ±10)	C000805	Integers in the form of square−25.	https://oeis.org/A098603
X[x] = −x*(x ±12)	Y[y] = y*(y ±12)	C000806	Integers in the form of square−36.	http://oeis.org/A098847
X[x] = −x*(x ±14)	Y[y] = y*(y ±14)	C000807	Integers in the form of square−49.	http://oeis.org/A098848
X[x] = −x*(x ±16)	Y[y] = y*(y ±16)	C000808	Integers in the form of square−64.	http://oeis.org/A098849
X[x] = −x*(x ±18)	Y[y] = y*(y ±18)	C000809	Integers in the form of square−81.	http://oeis.org/A098850
X[x] = −x*(x ±20)	Y[y] = y*(y ±20)	C000810	Integers in the form of square−100.	http://oeis.org/A120071
X[x] = −x*(x ±22)	Y[y] = y*(y ±22)	C000811	Integers in the form of square−121.	http://oeis.org/A132764
X[x] = −x*(x ±24)	Y[y] = y*(y ±24)	C000812	Integers in the form of square−144.	http://oeis.org/A132766
X[x] = −x*(x ±26)	Y[y] = y*(y ±26)	C000813	Integers in the form of square−169.	http://oeis.org/A132768
X[x] = −x*(x ±28)	Y[y] = y*(y ±28)	C000814	Integers in the form of square−196.	http://oeis.org/A132770
X[x] = −x*(x ±30)	Y[y] = y*(y ±30)	C000815	Integers in the form of square−225.	http://oeis.org/A132772

Figure 7. C000776 The (square sequence minus square number) table, further illustrating these sequences.

7 Theorem of the Sum of Even Numbers: A Visual Proof

Like the well-established theorem that the sum of odd numbers leads to square numbers, there is a parallel theorem for even numbers leading to oblong numbers. This can be demonstrated as follows (proof without words):



$$1 + 1 = 2$$
$$1 + 1 + 2 + 2 = 6$$
$$1 + 1 + 2 + 2 + 3 + 3 = 12$$
$$1 + 1 + 2 + 2 + 3 + 3 + 4 + 4 = 20$$

$$1 + 1 + 2 + 2 + 3 + 3 + 4 + 4 + 5 + 5 = 30$$
$$1 + 1 + 2 + 2 + 3 + 3 + 4 + 4 + 5 + 5 + 6 + 6 = 42$$
$$\ldots$$

Figure 8. C000900 The oblong numbers generator. The theorem of the partial sum of the even numbers.

The equivalent partial sums of positive even numbers show a similar pattern:

$$2 = 2$$
$$2 + 4 = 6$$
$$2 + 4 + 6 = 12$$
$$2 + 4 + 6 + 8 = 20$$
$$2 + 4 + 6 + 8 + 10 = 30$$
$$2 + 4 + 6 + 8 + 10 + 12 = 42$$
$$\ldots$$

Figure 9. C000900 The sequence of oblong numbers is the partial sum of the even numbers.

7.1 Partial Sum and Integral Analysis of Even Numbers

We derive the sequence of oblong numbers through the partial sum of the sequence of even numbers, expressed mathematically as:

$$y^2 + y = \sum_{1}^{y} 2y$$

This reveals that oblong numbers result from the partial sums of even numbers. Additionally, the difference between two consecutive oblong numbers is always even, and each even number is represented in this difference.

7.2 Reflecting about Goldbach´s Conjecture and Landau´s Problem with a Novel Approach

Note that the above formula plays a pivotal role in our approach to Goldbach's Conjecture and Landau's Problem. By adding a constant c to each term of this formula, we derive:

$$y^2 + y + c = \sum_{1}^{y} 2y + c$$

This adaptation implies that for each index position y, it's possible to find at least one value of c that results in two prime numbers. The first prime number is calculated using the formula $Prime_1 = y^2 + y + c$. The second prime number is derived from the formula $Prime_2 = (y \pm 1)^2 + y \pm 1 + c$. Specifically, for c values in the set {2, 3, 5, 11, 17, 41}, we obtain a sequence of numbers identified in the OEIS as https://oeis.org/A014556, which are known as Euler's 'Lucky' numbers."

Our hypothesis implies that specific constants, when added to our formula at distinct positions of y, can result in prime pairs. This systematic exploration of c values for various y could potentially uncover prime pairs that align with these famous mathematical conjectures, offering a new perspective on these long-standing mathematical challenges.

7.3 Visualizing the Approach:

Referencing C002209 The Oblong Parabolic PS[2,0,0] Sieve of Primes, we see that all vertical sequences conform to the form $Y[y] = y^2 + y + c$ for all integer values of y. Since $y^2 + y = y(y + 1)$ is always even, proving that there exists at least one odd c for any integer y that satisfies the prime conditions for $Prime_1$ and $Prime_2$ becomes crucial.

One approach is to initially show that all composite numbers are encompassed by at least two lines of Composite Generators (CGs), in this case quadratic CGs, while primes are covered by only one line. We can then use geometric reasoning to demonstrate the impossibility of all CG lines completely covering all integers more than once. The details and comprehensive proofs of these hypotheses can be found in [here].

The current literature on this topic appears to have gaps, particularly in not addressing even functions in relation to oblong numbers. This observation underscores the need for a broader approach in mathematical research, especially in exploring number theory and prime conjectures.

7.4 Geometric Patterns in Number Sequences

In Figure 8, we observe a fascinating geometric relationship: all triangles are part of a duplicated pair. When combined, each pair forms a rectangle, the area of which represents an oblong number given by the formula $y(y \pm 1)$. This product is always an even number, signifying a fundamental property of oblong numbers.

The oblong number sequence is a DES-type sequence, meaning each element is duplicated, no exception. By introducing an offset of one unit, we effectively add zero twice, allowing us to consider index $y \geq 0$. Therefore, the oblong number formula becomes:

$$y^2 + y = \sum_{0}^{y} 2y$$

In this case, index $y = 0$ is not an empty index. The true 'empty' index is found at the symmetry point between $y = 0$ and $y = -1$, specifically at $y = -0.5$. This is because both $y = 0$ and $y = -1$ yield two elements with 0 value.

If we apply the equation $y^2 - y$, the empty index shifts to lie between $y = 0$ and $y = 1$, at $y = 0.5$. This is due to both $y = 0$ and $y = 1$ producing two elements 0.

The partial sums of even numbers can be interpreted as partial integrals. To align the results of these integrals with the partial sums, we adjust the constant c (Eureka shift) generated in the polynomial integral. Observe:

$$\int (2y)dy = y^2 + c = square + c$$

This adjustment ensures that the integral results reflect oblong numbers, $y^2 - y$.

Between the oblong curve and the subsequent square curve, there is a Taylor shift of $h = 1/2$, directed along the Y-axis. Additionally, to align one curve with the other (shift-and-fit) [18], a Eureka shift [18] in the X-axis direction is necessary. Thus, we apply the Taylor shift of $h = 1/2$ to the integral of even numbers, along with a Eureka shift for $c = -h^2$ to shift in the X-axis direction:

Tally	Y	y^2	$c = -h^2$	$y^2 + c$
1	0.5	0.25	-0.25	0
2	1.5	2.25	-0.25	2
3	2.5	6.25	-0.25	6
4	3.5	12.25	-0.25	12
5	4.5	20.25	-0.25	20
6	5.5	30.25	-0.25	30
7	6.5	42.25	-0.25	42
8	7.5	56.25	-0.25	56
9	8.5	72.25	-0.25	72
10	9.5	90.25	-0.25	90
11	10.5	110.25	-0.25	110

Figure 10. C000900 Integral of the even numbers resulting in the oblong numbers.

This figure illustrates how the Eureka shift $c = -h^2$, derived from the Taylor shift of the square number by $h = -1/2$, affects the sequence.

In the special case where Taylor shift is half $h = -1/2$, we apply a horizontal Eureka shift for $c = -h^2 = -1/4$, leading to the alignment of the square and oblong sequences:

$Y[y] = y^2$ with shift h is:

$$Yº[y] = (y + h)^2 = y^2 + 2hy + h^2$$

For Taylor shift of half $h = -\frac{1}{2}$:

$$Yº[y] = \left(y - \frac{1}{2}\right)^2 = y^2 - y + \frac{1}{4}$$

Applying a horizontal Eureka shift for $c = -h^2 = -\frac{1}{4}$ then

$$Yº[y] = (y + h)^2 - h^2 = y^2 - y = oblong$$

Or we can say: $y^2 - \frac{1}{4}$ with a Taylor shift $h = -\frac{1}{2}$ is $\left(y - \frac{1}{2}\right)^2 - \frac{1}{4} = y^2 - y = oblong$

So, in offset $f = 0$:

$$oblong = y^2 - y = \int_{0.5}^{\frac{2y-1}{2}} (2y)dy = \int_{0.5}^{\frac{odd}{2}} (2y)dy$$

In offset $f = -1$:

$$oblong = y^2 + y = \int_{0.5}^{\frac{2y+1}{2}} (2y)dy = \int_{0.5}^{\frac{next\ odd}{2}} (2y)dy = \int_{0.5}^{y+0.5} (2y)dy$$

7.5 Applying Offsets in Partial Sums and Integral of the Even Numbers

When a single offset is applied to the partial sum of even numbers, the resulting sequence is:

$$4 = 4$$
$$4 + 6 = 10$$
$$4 + 6 + 8 = 18$$
$$4 + 6 + 8 + 10 = 28$$
$$4 + 6 + 8 + 10 + 12 = 40$$
$$\dots$$

These results are identified as the (oblong-2) numbers, corresponding to the sequence OEIS https://oeis.org/A028552. The mathematical representation is:

$$oblong - 2 = y^2 + y - 2 = \int_{1.5}^{\frac{2y+1}{2}} (2y)dy = \int_{1.5}^{\frac{next\ odd}{2}} (2y)dy = \int_{1.5}^{y+0.5} (2y)dy$$

Applying two offsets yields another interesting sequence:

$$6 = 6$$
$$6 + 8 = 14$$
$$6 + 8 + 10 = 24$$
$$6 + 8 + 10 + 12 = 36$$
$$\dots$$

This sequence results in the (oblong-6) numbers, found in OEIS https://oeis.org/A028557. Its formula is:

$$oblong - 6 = y^2 + y - 6 = \int_{2.5}^{\frac{2y+1}{2}} (2y)dy = \int_{2.5}^{y+0.5} (2y)dy$$

For any starting index $(n \pm 0.5)$, the relationship can be generalized as:

$$oblong - (n^2 \pm n) = y^2 \pm y - (n^2 \pm n) = \int_{n\pm0.5}^{\frac{2y\pm1}{2}} (2y)dy = \int_{n\pm0.5}^{y\pm0.5} (2y)dy$$

Because it is more common to work with definite integrals using integer limits, we can employ the Taylor shift by an amount of $h = \pm 0.5$ in the $Y[y] = 2y$ function, and we observe:

$$oblong - (n^2 \pm n) = y^2 \pm y - (n^2 \pm n) = \int_{n}^{y} (2y \pm 1)dy$$

This method reveals a variety of sequences in the form of (oblong sequence minus oblong number). These are visually represented in Figure 11, titled "C000777 Paraboctys $PS[-x^2 + x + 2, -x^2 + x, -x^2 + x]$" alongside a comprehensive table detailing each sequence and its corresponding OEIS link.

X-axis -->		-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y-axis -->	15	-30	0	28	54	78	100	120	138	154	168	180	190	198	204	208	210	210	208	204	198	190	180	168	154	138	120	100	78	54	28	0
	14	-58	-28	0	26	50	72	92	110	126	140	152	162	170	176	180	182	182	180	176	170	162	152	140	126	110	92	72	50	26	0	-28
	13	-84	-54	-26	0	24	46	66	84	100	114	126	136	144	150	154	156	156	154	150	144	136	126	114	100	84	66	46	24	0	-26	-54
	12	-108	-78	-50	-24	0	22	42	60	76	90	102	112	120	126	130	132	132	130	126	120	112	102	90	76	60	42	22	0	-24	-50	-78
	11	-130	-100	-72	-46	-22	0	20	38	54	68	80	90	98	104	108	110	110	108	104	98	90	80	68	54	38	20	0	-22	-46	-72	-100
	10	-150	-120	-92	-66	-42	-20	0	18	34	48	60	70	78	84	88	90	90	88	84	78	70	60	48	34	18	0	-20	-42	-66	-92	-120
	9	-168	-138	-110	-84	-60	-38	-18	0	16	30	42	52	60	66	70	72	72	70	66	60	52	42	30	16	0	-18	-38	-60	-84	-110	-138
	8	-184	-154	-126	-100	-76	-54	-34	-16	0	14	26	36	44	50	54	56	56	54	50	44	36	26	14	0	-16	-34	-54	-76	-100	-126	-154
	7	-198	-168	-140	-114	-90	-68	-48	-30	-14	0	12	22	30	36	40	42	42	40	36	30	22	12	0	-14	-30	-48	-68	-90	-114	-140	-168
	6	-210	-180	-152	-126	-102	-80	-60	-42	-26	-12	0	10	18	24	28	30	30	28	24	18	10	0	-12	-26	-42	-60	-80	-102	-126	-152	-180
	5	-220	-190	-162	-136	-112	-90	-70	-52	-36	-22	-10	0	8	14	18	20	20	18	14	8	0	-10	-22	-36	-52	-70	-90	-112	-136	-162	-190
	4	-228	-198	-170	-144	-120	-98	-78	-60	-44	-30	-18	-8	0	6	10	12	12	10	6	0	-8	-18	-30	-44	-60	-78	-98	-120	-144	-170	-198
	3	-234	-204	-176	-150	-126	-104	-84	-66	-50	-36	-24	-14	-6	0	4	6	6	4	0	-6	-14	-24	-36	-50	-66	-84	-104	-126	-150	-176	-204
	2	-238	-208	-180	-154	-130	-108	-88	-70	-54	-40	-28	-18	-10	-4	0	2	2	0	-4	-10	-18	-28	-40	-54	-70	-88	-108	-130	-154	-180	-208
Y[1]	1	-240	-210	-182	-156	-132	-110	-90	-72	-56	-42	-30	-20	-12	-6	-2	0	0	-2	-6	-12	-20	-30	-42	-56	-72	-90	-110	-132	-156	-182	-210
Y[0]	0	-240	-210	-182	-156	-132	-110	-90	-72	-56	-42	-30	-20	-12	-6	-2	0	0	-2	-6	-12	-20	-30	-42	-56	-72	-90	-110	-132	-156	-182	-210
Y[-1]	-1	-238	-208	-180	-154	-130	-108	-88	-70	-54	-40	-28	-18	-10	-4	0	2	2	0	-4	-10	-18	-28	-40	-54	-70	-88	-108	-130	-154	-180	-208
	-2	-234	-204	-176	-150	-126	-104	-84	-66	-50	-36	-24	-14	-6	0	4	6	6	4	0	-6	-14	-24	-36	-50	-66	-84	-104	-126	-150	-176	-204
	-3	-228	-198	-170	-144	-120	-98	-78	-60	-44	-30	-18	-8	0	6	10	12	12	10	6	0	-8	-18	-30	-44	-60	-78	-98	-120	-144	-170	-198
	-4	-220	-190	-162	-136	-112	-90	-70	-52	-36	-22	-10	0	8	14	18	20	20	18	14	8	0	-10	-22	-36	-52	-70	-90	-112	-136	-162	-190
	-5	-210	-180	-152	-126	-102	-80	-60	-42	-26	-12	0	10	18	24	28	30	30	28	24	18	10	0	-12	-26	-42	-60	-80	-102	-126	-152	-180
	-6	-198	-168	-140	-114	-90	-68	-48	-30	-14	0	12	22	30	36	40	42	42	40	36	30	22	12	0	-14	-30	-48	-68	-90	-114	-140	-168
	-7	-184	-154	-126	-100	-76	-54	-34	-16	0	14	26	36	44	50	54	56	56	54	50	44	36	26	14	0	-16	-34	-54	-76	-100	-126	-154
	-8	-168	-138	-110	-84	-60	-38	-18	0	16	30	42	52	60	66	70	72	72	70	66	60	52	42	30	16	0	-18	-38	-60	-84	-110	-138
	-9	-150	-120	-92	-66	-42	-20	0	18	34	48	60	70	78	84	88	90	90	88	84	78	70	60	48	34	18	0	-20	-42	-66	-92	-120
	-10	-130	-100	-72	-46	-22	0	20	38	54	68	80	90	98	104	108	110	110	108	104	98	90	80	68	54	38	20	0	-22	-46	-72	-100
	-11	-108	-78	-50	-24	0	22	42	60	76	90	102	112	120	126	130	132	132	130	126	120	112	102	90	76	60	42	22	0	-24	-50	-78
	-12	-84	-54	-26	0	24	46	66	84	100	114	126	136	144	150	154	156	156	154	150	144	136	126	114	100	84	66	46	24	0	-26	-54
	-13	-58	-28	0	26	50	72	92	110	126	140	152	162	170	176	180	182	182	180	176	170	162	152	140	126	110	92	72	50	26	0	-28
	-14	-30	0	28	54	78	100	120	138	154	168	180	190	198	204	208	210	210	208	204	198	190	180	168	154	138	120	100	78	54	28	0
	-15	0	30	58	84	108	130	150	168	184	198	210	220	228	234	238	240	240	238	234	228	220	210	198	184	168	150	130	108	84	58	30

Figure 11. C000777 Paraboctys $PS[-x^2 + x + 2, -x^2 + x, -x^2 + x]$ The (oblong sequence minus the oblong number) sequences.

Horizontals	Verticals	SNYPO	Description	OEIS
X[x] = –x*(x ±1)	Y[y] = y*(y ±1)	C000900	Integers in the form of oblong–0.	http://oeis.org/A002378
X[x] = –x*(x ±3)	Y[y] = y*(y ±3)	C000901	Integers in the form of oblong–2.	http://oeis.org/A028552
X[x] = –x*(x ±5)	Y[y] = y*(y ±5)	C000902	Integers in the form of oblong–6.	http://oeis.org/A028557
X[x] = –x*(x ±7)	Y[y] = y*(y ±7)	C000903	Integers in the form of oblong–12.	http://oeis.org/A028563
X[x] = –x*(x ±9)	Y[y] = y*(y ±9)	C000904	Integers in the form of oblong–20.	http://oeis.org/A028569
X[x] = –x*(x ±11)	Y[y] = y*(y ±11)	C000905	Integers in the form of oblong–30.	http://oeis.org/A119412
X[x] = –x*(x ±13)	Y[y] = y*(y ±13)	C000906	Integers in the form of oblong–42.	http://oeis.org/A132759
X[x] = –x*(x ±15)	Y[y] = y*(y ±15)	C000907	Integers in the form of oblong–56.	http://oeis.org/A132760
X[x] = –x*(x ±17)	Y[y] = y*(y ±17)	C000908	Integers in the form of oblong–72.	http://oeis.org/A132761
X[x] = –x*(x ±19)	Y[y] = y*(y ±19)	C000909	Integers in the form of oblong–90.	http://oeis.org/A132762
X[x] = –x*(x ±21)	Y[y] = y*(y ±21)	C000910	Integers in the form of oblong–110.	http://oeis.org/A132763
X[x] = –x*(x ±23)	Y[y] = y*(y ±23)	C000911	Integers in the form of oblong–132.	http://oeis.org/A132765
X[x] = –x*(x ±25)	Y[y] = y*(y ±25)	C000912	Integers in the form of oblong–156.	http://oeis.org/A132767
X[x] = –x*(x ±27)	Y[y] = y*(y ±27)	C000913	Integers in the form of oblong–182.	http://oeis.org/A132769
X[x] = –x*(x ±29)	Y[y] = y*(y ±29)	C000914	Integers in the form of oblong–210.	http://oeis.org/A132771
X[x] = –x*(x ±31)	Y[y] = y*(y ±31)	C000915	Integers in the form of oblong–240.	http://oeis.org/A132773

Figure 12. C000777 The (oblong sequence minus oblong number) sequences table.

7.6 Conclusions

These numbers, generated through specific methods involving partial sums and integrals, reveal intricate patterns and relationships within number theory. When we combine the concepts from Figure 1 and Figure 2, we observe interesting angular relationships between square and oblong numbers, influenced by Taylor shifts.

Taylor Shift of $\mathbf{0.5}$: For a Taylor shift of $h = 0.5$, there is a 45º angle between the square and oblong sequences. This geometric relationship provides a visual representation of the mathematical connection between these sequences.

Taylor Shift of 1 Unit: When applying a Taylor shift of $h = 1$, we find a 90º angle between the square and (square-1) sequences. This demonstrates a distinct mathematical relationship where each sequence shift results in a perpendicular orientation.

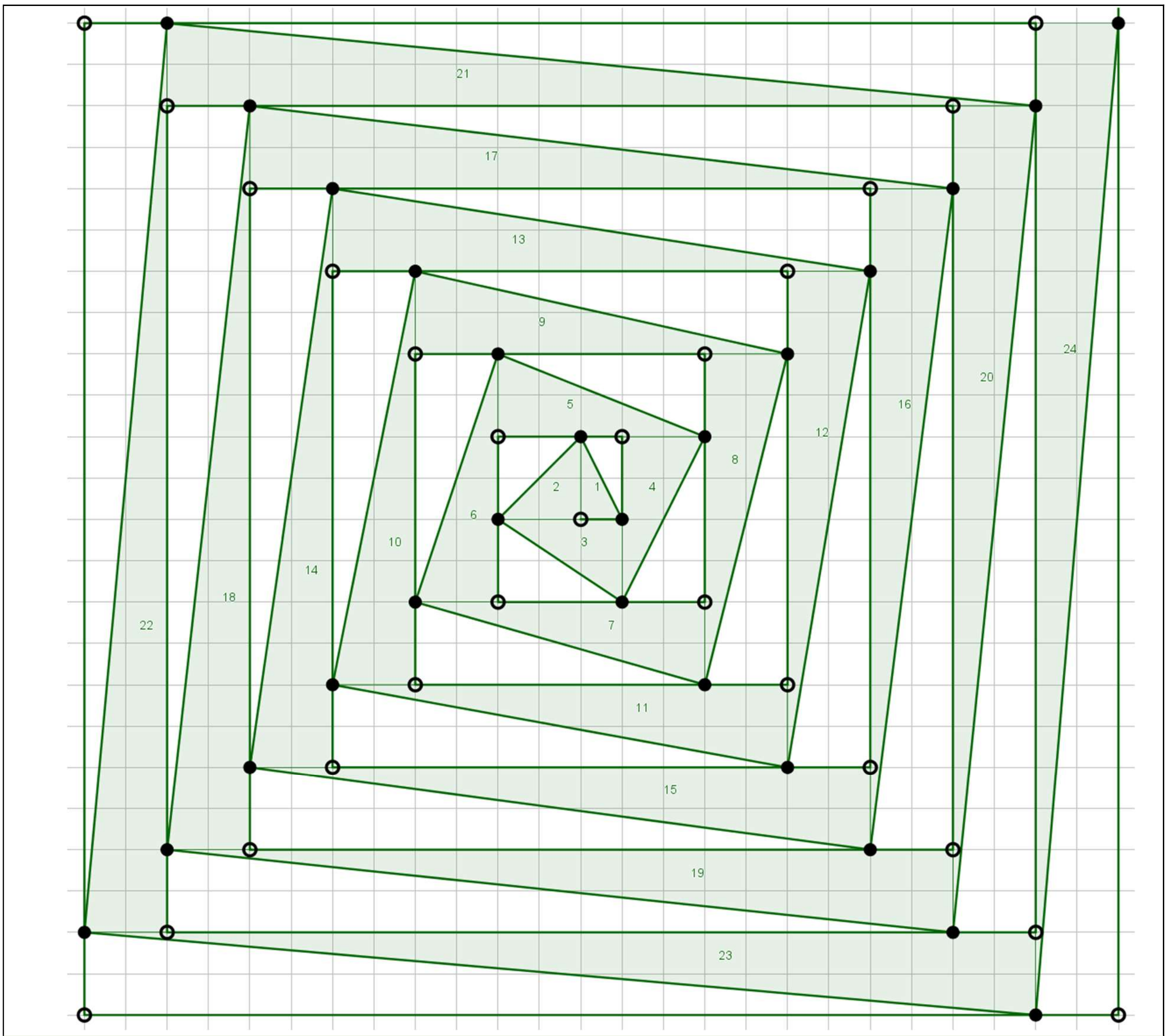
For every Taylor shift of 0.5 between a sequence of (square sequence minus square number) and the subsequent sequence of (oblong sequence minus oblong number), there exists a 45-degree angle.

For every offset of 1 (or Taylor shift of 1) between one sequence of (square sequence minus square number) and the next, and between one sequence of (oblong sequence minus oblong number) and the next, there is a 90-degree angle.

These observations align with the findings and discussions in the Mersenne Forum thread., as well as Mersenne Forum thread.

8 Exploring the Theorem of the Sum of Positive Integer Numbers

The relationship between oblong and triangular numbers is a fundamental concept in number theory. This relationship can be visualized as half of an oblong number equating to a triangular number. Consider the following visual demonstration using green triangle areas:



$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10$$
$$1 + 2 + 3 + 4 + 5 = 15$$
$$1 + 2 + 3 + 4 + 5 + 6 = 21$$
$$\dots$$

Figure 13. C000899 https://oeis.org/A000217 The Triangular Numbers Generator and Theorem of the Sum of Positive Integer Numbers.

8.1 Partial Sum and Integral of the Integer Numbers

Like oblongs, triangular numbers can be derived through various summation methods. From 1 to y, or 1 to $y - 1$:

$$triangular\ numbers = \frac{y^2 + y}{2} = \sum_{1}^{y} y$$

$$triangular\ numbers = \frac{y^2 - y}{2} = \sum_{1}^{y-1} y$$

From 0 to y, or 0 to $y - 1$:

$$triangular\ numbers = \frac{y^2 + y}{2} = \sum_{0}^{y} y$$

$$triangular\ numbers = \frac{y^2 - y}{2} = \sum_{0}^{y-1} y$$

In these demonstrations, we accept $y \geq 0$, acknowledging that at $y = 0$ is not an empty index but a valid index to generate an element. The empty index, reflecting symmetry, is located at $y = -0.5$. It is a DES-type sequence equal oblongs.

We can interpret these sums as partial integrals, requiring adjustment of the constant c:

$$\int (y)dy = \frac{y^2}{2} + c = \frac{square}{2} + c$$

However, applying a Taylor shift of $h = \pm 0.5$ in $Y[y] = y$, we observe:

$$\int (y \pm 0.5)dy = \frac{y^2 \pm y}{2} + c = triangular + c$$

In definite integrals we have:

$$\int_0^y (y + 0.5) = \frac{y^2 + y}{2} - \frac{0^2 + 0}{2} = \frac{y^2 + y}{2} - 0$$

$$\int_1^y (y + 0.5) = \frac{y^2 + y}{2} - \frac{1^2 + 1}{2} = \frac{y^2 + y}{2} - 1$$

$$\int_2^y (y + 0.5) = \frac{y^2 + y}{2} - \frac{2^2 + 2}{2} = \frac{y^2 + y}{2} - 3$$

Generalizing:

$$\int_n^y (y + 0.5) = \frac{y^2 + y}{2} - \frac{n^2 + n}{2}$$

Figure 14, titled C001680 Paraboctys $PS[-0.5x^2 + 0.5x + 1, -0.5x^2 + 0.5x, -0.5x^2 + 0.5x]$ provides a visual representation of the (triangular sequence minus triangular number).

	X-axi	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y-axis -->	15	-15	0	14	27	39	50	60	69	77	84	90	95	99	102	104	105	105	104	102	99	95	90	84	77	69	60	50	39	27	14	0
	14	-29	-14	0	13	25	36	46	55	63	70	76	81	85	88	90	91	91	90	88	85	81	76	70	63	55	46	36	25	13	0	-14
	13	-42	-27	-13	0	12	23	33	42	50	57	63	68	72	75	77	78	78	77	75	72	68	63	57	50	42	33	23	12	0	-13	-27
	12	-54	-39	-25	-12	0	11	21	30	38	45	51	56	60	63	65	66	66	65	63	60	56	51	45	38	30	21	11	0	-12	-25	-39
	11	-65	-50	-36	-23	-11	0	10	19	27	34	40	45	49	52	54	55	55	54	52	49	45	40	34	27	19	10	0	-11	-23	-36	-50
	10	-75	-60	-46	-33	-21	-10	0	9	17	24	30	35	39	42	44	45	45	44	42	39	35	30	24	17	9	0	-10	-21	-33	-46	-60
	9	-84	-69	-55	-42	-30	-19	-9	0	8	15	21	26	30	33	35	36	36	35	33	30	26	21	15	8	0	-9	-19	-30	-42	-55	-69
	8	-92	-77	-63	-50	-38	-27	-17	-8	0	7	13	18	22	25	27	28	28	27	25	22	18	13	7	0	-8	-17	-27	-38	-50	-63	-77
	7	-99	-84	-70	-57	-45	-34	-24	-15	-7	0	6	11	15	18	20	21	21	20	18	15	11	6	0	-7	-15	-24	-34	-45	-57	-70	-84
	6	-105	-90	-76	-63	-51	-40	-30	-21	-13	-6	0	5	9	12	14	15	15	14	12	9	5	0	-6	-13	-21	-30	-40	-51	-63	-76	-90
	5	-110	-95	-81	-68	-56	-45	-35	-26	-18	-11	-5	0	4	7	9	10	10	9	7	4	0	-5	-11	-18	-26	-35	-45	-56	-68	-81	-95
	4	-114	-99	-85	-72	-60	-49	-39	-30	-22	-15	-9	-4	0	3	5	6	6	5	3	0	-4	-9	-15	-22	-30	-39	-49	-60	-72	-85	-99
	3	-117	-102	-88	-75	-63	-52	-42	-33	-25	-18	-12	-7	-3	0	2	3	3	2	0	-3	-7	-12	-18	-25	-33	-42	-52	-63	-75	-88	-102
	2	-119	-104	-90	-77	-65	-54	-44	-35	-27	-20	-14	-9	-5	-2	0	1	1	0	-2	-5	-9	-14	-20	-27	-35	-44	-54	-65	-77	-90	-104
Y[1]	1	-120	-105	-91	-78	-66	-55	-45	-36	-28	-21	-15	-10	-6	-3	-1	0	0	-1	-3	-6	-10	-15	-21	-28	-36	-45	-55	-66	-78	-91	-105
Y[0]	0	-120	-105	-91	-78	-66	-55	-45	-36	-28	-21	-15	-10	-6	-3	-1	0	0	-1	-3	-6	-10	-15	-21	-28	-36	-45	-55	-66	-78	-91	-105
Y[-1]	-1	-119	-104	-90	-77	-65	-54	-44	-35	-27	-20	-14	-9	-5	-2	0	1	1	0	-2	-5	-9	-14	-20	-27	-35	-44	-54	-65	-77	-90	-104
	-2	-117	-102	-88	-75	-63	-52	-42	-33	-25	-18	-12	-7	-3	0	2	3	3	2	0	-3	-7	-12	-18	-25	-33	-42	-52	-63	-75	-88	-102
	-3	-114	-99	-85	-72	-60	-49	-39	-30	-22	-15	-9	-4	0	3	5	6	6	5	3	0	-4	-9	-15	-22	-30	-39	-49	-60	-72	-85	-99
	-4	-110	-95	-81	-68	-56	-45	-35	-26	-18	-11	-5	0	4	7	9	10	10	9	7	4	0	-5	-11	-18	-26	-35	-45	-56	-68	-81	-95
	-5	-105	-90	-76	-63	-51	-40	-30	-21	-13	-6	0	5	9	12	14	15	15	14	12	9	5	0	-6	-13	-21	-30	-40	-51	-63	-76	-90
	-6	-99	-84	-70	-57	-45	-34	-24	-15	-7	0	6	11	15	18	20	21	21	20	18	15	11	6	0	-7	-15	-24	-34	-45	-57	-70	-84
	-7	-92	-77	-63	-50	-38	-27	-17	-8	0	7	13	18	22	25	27	28	28	27	25	22	18	13	7	0	-8	-17	-27	-38	-50	-63	-77
	-8	-84	-69	-55	-42	-30	-19	-9	0	8	15	21	26	30	33	35	36	36	35	33	30	26	21	15	8	0	-9	-19	-30	-42	-55	-69
	-9	-75	-60	-46	-33	-21	-10	0	9	17	24	30	35	39	42	44	45	45	44	42	39	35	30	24	17	9	0	-10	-21	-33	-46	-60
	-10	-65	-50	-36	-23	-11	0	10	19	27	34	40	45	49	52	54	55	55	54	52	49	45	40	34	27	19	10	0	-11	-23	-36	-50
	-11	-54	-39	-25	-12	0	11	21	30	38	45	51	56	60	63	65	66	66	65	63	60	56	51	45	38	30	21	11	0	-12	-25	-39
	-12	-42	-27	-13	0	12	23	33	42	50	57	63	68	72	75	77	78	78	77	75	72	68	63	57	50	42	33	23	12	0	-13	-27
	-13	-29	-14	0	13	25	36	46	55	63	70	76	81	85	88	90	91	91	90	88	85	81	76	70	63	55	46	36	25	13	0	-14
	-14	-15	0	14	27	39	50	60	69	77	84	90	95	99	102	104	105	105	104	102	99	95	90	84	77	69	60	50	39	27	14	0
	-15	0	15	29	42	54	65	75	84	92	99	105	110	114	117	119	120	120	119	117	114	110	105	99	92	84	75	65	54	42	29	15

Figure 14. C001680 Paraboctys $PS[-0.5x^2 + 0.5x + 1, -0.5x^2 + 0.5x, -0.5x^2 + 0.5x]$.The (triangular sequence minus triangular number) sequences.

Horizontals	Verticals	SNYPO	Description	OEIS
$X[x] = -x * (x \pm 1)/2$	$Y[y] = y * (y \pm 1)/2$		Integers in the form of triangular–0.	https://oeis.org/A000217
$X[x] = -x * (x \pm 3)/2$	$Y[y] = y * (y \pm 3)/2$		Integers in the form of triangular–1.	https://oeis.org/A000096
$X[x] = -x * (x \pm 5)/2$	$Y[y] = y * (y \pm 5)/2$		Integers in the form of triangular–3.	https://oeis.org/A055998
$X[x] = -x * (x \pm 7)/2$	$Y[y] = y * (y \pm 7)/2$		Integers in the form of triangular–6.	https://oeis.org/A055999
$X[x] = -x * (x \pm 9)/2$	$Y[y] = y * (y \pm 9)/2$		Integers in the form of triangular–10.	https://oeis.org/A056000
$X[x] = -x * (x \pm 11)/2$	$Y[y] = y * (y \pm 11)/2$		Integers in the form of triangular–15.	https://oeis.org/A056115
$X[x] = -x * (x \pm 13)/2$	$Y[y] = y * (y \pm 13)/2$		Integers in the form of triangular–21.	https://oeis.org/A056119
$X[x] = -x * (x \pm 15)/2$	$Y[y] = y * (y \pm 15)/2$		Integers in the form of triangular–28.	https://oeis.org/A056121
$X[x] = -x * (x \pm 17)/2$	$Y[y] = y * (y \pm 17)/2$		Integers in the form of triangular–36.	https://oeis.org/A056126
$X[x] = -x * (x \pm 19)/2$	$Y[y] = y * (y \pm 19)/2$		Integers in the form of triangular–45.	https://oeis.org/A051942
$X[x] = -x * (x \pm 21)/2$	$Y[y] = y * (y \pm 21)/2$		Integers in the form of triangular–55.	https://oeis.org/A101859
$X[x] = -x * (x \pm 23)/2$	$Y[y] = y * (y \pm 23)/2$		Integers in the form of triangular–66.	https://oeis.org/A132754
$X[x] = -x * (x \pm 25)/2$	$Y[y] = y * (y \pm 25)/2$		Integers in the form of triangular–78.	https://oeis.org/A132755
$X[x] = -x * (x \pm 27)/2$	$Y[y] = y * (y \pm 27)/2$		Integers in the form of triangular–91.	
$X[x] = -x * (x \pm 29)/2$	$Y[y] = y * (y \pm 29)/2$		Integers in the form of triangular–105.	
$X[x] = -x * (x \pm 31)/2$	$Y[y] = y * (y \pm 31)/2$		Integers in the form of triangular–120.	

Figure 15. C001680 The (triangular sequence minus triangular number) sequences table.

9 Understanding the 3 Basic Quadratic Theorems

The three basic quadratic theorems start from the concept of zero area, and the fundamental notion that allows us to express any area as an integral. The origin of zero area derives from the fact that the partial integral of a unit, $\int_f^y 1\,dy$, equals $y - f$. This implies that for any integer offset f and integer index y, $\int_f^y 1\,dy$ results in sequences of integers in the form of an entire sequence minus one integer from that sequence.

Generalizing, for any offset f in the integral $\int_f^y Y[y]dy$, we end up with $Yº[y] - Yº[f]$, where the final sequence always takes the form of (the sequence of numbers $Yº[y]$ minus an integer $Yº[f]$). Therefore, the resulting sequence will always have at least one zero element, and the number of zero elements is at most equal to the polynomial degree of the function resulting from the integral.

Considering the specific case of $Y[y] = y$, we have $\int_f^y y\,dy\ = \frac{y^2-f^2}{2}$. For sequences in the form of (square sequence minus square number), the integral is $\int_f^y 2y\,dy\ =\ y^2 - f^2$. Applying a Taylor shift of h to the function $Y[y] = 2y$, we get $\int_f^y 2(y+h)dy =\ (y^2 + 2hy) - (f^2 + 2hf) =\ y(y + 2h) - f(f + 2h)$. For the terms of this equation to continue generating integers, $2h$ needs to be an integer, implying that for each Taylor shift of $h = 0.5$, we will get zero in some index of the sequence.

Applying a Taylor shift h to the function of odd numbers, we have $\int_f^y (2(y+h) - 1)dy = (y^2 - y + 2hy) - (f^2 - f + 2hf) =\ y(y + 2h - 1) - f(f + 2h - 1)$, where $(2h - 1)$ needs to be an integer. This also occurs only when the Taylor shift is a multiple of $h = 0.5$.

For triangular numbers, we keep $\int_f^y y\,dy\ = \frac{y^2-f^2}{2}$ and apply the Taylor shift, resulting in $\int_f^y (y+h)dy = \frac{(y^2+2hy)-(f^2+2hf)}{2} = \frac{y(y+2h)-f(f+2h)}{2} = \frac{y^2-f^2}{2} +\ h(y - f)$, where we obtain sequences of integers with Taylor shift being a multiple of $h = 0.5$.

9.1 Investigation of the 2nd Degree Coefficient a in Quadratics

Triangular numbers result from $\int_f^y (y+h)dy$, with a 2nd degree coefficient $a = 1/2$. Oblong numbers come from $\int_f^y 2(y+h)dy$, with $a = 1$. Square numbers come from $\int_f^y (2(y+h) - 1)dy$, with $a = 1$ as well.

This suggests a continuation of this integral series:

$$\int_f^y 3(y+h)dy = \frac{3y^2 + 6hy}{2} - \frac{3f^2 + 6hf}{2}$$

$$\int_f^y (3(y+h) - 1)\,dy = \frac{3y^2 + 6hy - 2y}{2} - \frac{3f^2 + 6hf - 2f}{2}$$

$$\int_f^y (3(y+h)-2)\, dy = \frac{3y^2+6hy-4y}{2} - \frac{3f^2+6hf-4f}{2}$$

$$\int_f^y 4(y+h)dy = (2y^2+4hy) - (2f^2+4hf)$$

$$\int_f^y (4(y+h)-1)\, dy = (2y^2-y+4hy) - (2f^2-f+4hf)$$

$$\int_f^y (4(y+h)-2)\, dy = (2y^2-2y+4hy) - (2f^2-2f+4hf)$$

$$\int_f^y (4(y+h)-3)\, dy = (2y^2-3y+4hy) - (2f^2-3f+4hf)$$

and so on. Generically, for $0 \le k < a$:

$$\int_f^y (a(y+h)-k)dy = \frac{ay^2+a(2h)y-2ky}{2} - \frac{af^2+a(2h)f-2kf}{2}$$

9.2 Defining the $Y2[y] - Y2[f]$ Sequence in Quadratic Polynomial Analysis

Let us consider the $Y2[y] - Y2[f]$ sequence defined by the quadratic function $Y2$ and evaluated at two distinct points, y and f. This sequence can be expressed as the difference between the quadratic polynomial evaluations at these points, represented by the integral:

$$Y2[y] - Y2[f] = \int_f^y (a(y+h)-k)\, dy = \frac{ay^2+a(2h)y-2ky}{2} - \frac{af^2+a(2h)f-2kf}{2}$$

It is important to note that '2' in $Y2[y]$ denotes the polynomial's degree, indicating a quadratic function.

Zero Element in the Sequence:

A notable property of the $Y2[y] - Y2[f]$ sequence is its inherent structure: it always forms a sequence of quadratic expressions minus an element from the same set. Therefore, each $Y2[y] - Y2[f]$ sequence invariably contains at least one zero element. This leads to the possibility of expressing the sequence in the form of a trio $\{x_1, 0, x_3\}$, where some offset exists that aligns with the zero element.

Application of Taylor Shifts:

When applying a Taylor shift, specifically multiples of $h = 0.5$, the coefficient $2h$ remains an integer. This is crucial in maintaining the integrity of the sequence's integer properties.

Analysis of the Simplest Quadratic Equation:

Referencing the study 'Shift, Symmetry, and Asymmetry in Polynomial Sequences' [18], we consider the simplest form of a quadratic equation:

$$x = Y2[y] = ay^2 + by + c = \{x_1, x_2, x_3\} = \frac{x_1 - 2x_2 + x_3}{2}y^2 + \frac{x_3 - x_2}{2}y + x_2$$

Here, the second-degree coefficient 'a' is derived as:

$$a = \frac{x_1 - 2x_2 + x_3}{2}$$

Given that any sequence $\int_f^y Y[y]dy = Yº[y] - Yº[f]$ contains at least one zero element, and this zero can assume any offset position without altering the value of 'a', we can simplify the calculation of 'a' by considering $x_2 = 0$:

$$a = \frac{x_1 + x_3}{2}$$

Alternatively, we can express this as $x_1 + x_2 = 2a$.

Case Studies of Coefficient 'a':

For a coefficient $a = 0.5$, we find $x_1 + x_2 = 1$, leading to possible sequences $x = Y2[y] = ay\hat{}2 + by + c = \{1,0,0\}$ or $\{0,0,1\}$. Similarly, for $a = 1$, $x_1 + x_2 = 2$, yielding sequences such as $\{1,0,1\}$, $\{2,0,0\}$, or $\{0,0,2\}$, and so forth.

Mappings and Eureka Shifts:

These properties allow us to map all quadratic sequences in the form of $Y2[y] - Y2[f]$. By applying the Eureka shift to each distinct sequence, we can obtain any desired quadratic sequence.

9.3 Quadratic CG (Composite Generator) Concept:

The term "Quadratic CG" (Composite Generator) is coined to describe the $Y2[y] - Y2[f]$ sequences. This concept encapsulates the sequence's inherent ability to yield either none or a finite number of prime numbers while simultaneously producing an infinite series of composite numbers. The Composite Generator's notion underscores the sequence's versatility and its potential applications in number theory.

10 C003109 Atlas of Quadratic Composite Generators (CGs): A Comprehensive Compilation

Overview of the Atlas:

The "C003109 Atlas of Quadratic Composite Generators" presents a meticulous aggregation of all possible three-element integer sequences that form quadratic Composite Generators (CGs). This atlas is a significant resource for researchers delving into the intricate patterns and properties of polynomial integer sequences.

10.1 Atlas Structure and Content:

Section on Trio Form CGs (Figure 16): This section visualizes the quadratic CGs in trio form, providing an intuitive understanding of their structural composition.

a	2a = x_1+x_3	{2a,0,0}	{2a−1,0,1}	{2a−2,0,2}	{2a−3,0,3}	{2a−4,0,4}	{2a−5,0,5}	{2a−6,0,6}	{2a−7,0,7}	{2a−8,0,8}	{2a−9,0,9}	{2a−10,0,10}
0.5	1	{1,0,0}	{0,0,1}	{−1,0,2}	{−2,0,3}	{−3,0,4}	{−4,0,5}	{−5,0,6}	{−6,0,7}	{−7,0,8}	{−8,0,9}	{−9,0,10}
1	2	{2,0,0}	{1,0,1}	{0,0,2}	{−1,0,3}	{−2,0,4}	{−3,0,5}	{−4,0,6}	{−5,0,7}	{−6,0,8}	{−7,0,9}	{−8,0,10}
1.5	3	{3,0,0}	{2,0,1}	{1,0,2}	{0,0,3}	{−1,0,4}	{−2,0,5}	{−3,0,6}	{−4,0,7}	{−5,0,8}	{−6,0,9}	{−7,0,10}
2	4	{4,0,0}	{3,0,1}	{2,0,2}	{1,0,3}	{0,0,4}	{−1,0,5}	{−2,0,6}	{−3,0,7}	{−4,0,8}	{−5,0,9}	{−6,0,10}
2.5	5	{5,0,0}	{4,0,1}	{3,0,2}	{2,0,3}	{1,0,4}	{0,0,5}	{−1,0,6}	{−2,0,7}	{−3,0,8}	{−4,0,9}	{−5,0,10}
3	6	{6,0,0}	{5,0,1}	{4,0,2}	{3,0,3}	{2,0,4}	{1,0,5}	{0,0,6}	{−1,0,7}	{−2,0,8}	{−3,0,9}	{−4,0,10}
3.5	7	{7,0,0}	{6,0,1}	{5,0,2}	{4,0,3}	{3,0,4}	{2,0,5}	{1,0,6}	{0,0,7}	{−1,0,8}	{−2,0,9}	{−3,0,10}
4	8	{8,0,0}	{7,0,1}	{6,0,2}	{5,0,3}	{4,0,4}	{3,0,5}	{2,0,6}	{1,0,7}	{0,0,8}	{−1,0,9}	{−2,0,10}
4.5	9	{9,0,0}	{8,0,1}	{7,0,2}	{6,0,3}	{5,0,4}	{4,0,5}	{3,0,6}	{2,0,7}	{1,0,8}	{0,0,9}	{−1,0,10}
5	10	{10,0,0}	{9,0,1}	{8,0,2}	{7,0,3}	{6,0,4}	{5,0,5}	{4,0,6}	{3,0,7}	{2,0,8}	{1,0,9}	{0,0,10}

Figure 16. C003102 Atlas of Quadratic CGs in the form of Trios.

Integer Sequence Formulation (Figure 17): Transitioning from trio forms, the atlas illustrates the corresponding integer sequences of quadratic CGs. A high-resolution version of Figure 17 is available via [link], offering detailed insights into sequence arrangements.

	Tally	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	x_focu	0.38	0	0.25	-0.2	0.13	-0.4	0	0.13	-0.5	-0.1	0.08	-0.7	-0.3	0	0.08	-0.8	-0.4	-0.1	0.05	-0.9	-0.5	-0.2	0	0.06
	LR	2	1	1	0.67	0.67	0.5	0.5	0.5	0.4	0.4	0.4	0.33	0.33	0.33	0.33	0.29	0.29	0.29	0.29	0.25	0.25	0.25	0.25	0.25
	Δ	0.25	1	0	2.25	0.25	4	1	0	6.25	2.25	0.25	9	4	1	0	12.3	6.25	2.25	0.25	16	9	4	1	0
	√Δ	0.5	1	0	1.5	0.5	2	1	0	2.5	1.5	0.5	3	2	1	0	3.5	2.5	1.5	0.5	4	3	2	1	0
	C. G.	0.5	0	0	0.5	0.5	0	0	0	0.5	0.5	0.5	0	0	0	0	0.5	0.5	0.5	0.5	0	0	0	0	0
	R1	1	1	0	1	0.33	1	0.5	0	1	0.6	0.2	1	0.67	0.33	0	1	0.71	0.43	0.14	1	0.75	0.5	0.25	0
	R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	R2−R1	-1	-1	0	-1	-0.3	-1	-0.5	0	-1	-0.6	-0.2	-1	-0.7	-0.3	0	-1	-0.7	-0.4	-0.1	-1	-0.8	-0.5	-0.3	0
	R2+R1	1	1	0	1	0.33	1	0.5	0	1	0.6	0.2	1	0.67	0.33	0	1	0.71	0.43	0.14	1	0.75	0.5	0.25	0
	R2*R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	x_sp	-0.1	-0.3	0	-0.4	-0	-0.5	-0.1	0	-0.6	-0.2	-0	-0.8	-0.3	-0.1	0	-0.9	-0.4	-0.2	-0	-1	-0.6	-0.3	-0.1	0
	y_sp	0.5	0.5	0	0.5	0.17	0.5	0.25	0	0.5	0.3	0.1	0.5	0.33	0.17	0	0.5	0.36	0.21	0.07	0.5	0.38	0.25	0.13	0
	f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	type	DES	DES	SUB	DES	ACC	DES	ACC	SUB	DES	ACC	ACC	DES	ACC	ACC	SUB	DES	ACC	ACC	ACC	DES	ACC	ACC	ACC	SUB
	a	0.5	1	1	1.5	1.5	2	2	2	2.5	2.5	2.5	3	3	3	3	3.5	3.5	3.5	3.5	4	4	4	4	4
	b	-0.5	-1	0	-1.5	-0.5	-2	-1	0	-2.5	-1.5	-0.5	-3	-2	-1	0	-3.5	-2.5	-1.5	-0.5	-4	-3	-2	-1	0
	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	45	90	100	135	145	180	190	200	225	235	245	270	280	290	300	315	325	335	345	360	370	380	390	400
	9	36	72	81	108	117	144	153	162	180	189	198	216	225	234	243	252	261	270	279	288	297	306	315	324
	8	28	56	64	84	92	112	120	128	140	148	156	168	176	184	192	196	204	212	220	224	232	240	248	256
	7	21	42	49	63	70	84	91	98	105	112	119	126	133	140	147	147	154	161	168	168	175	182	189	196
	6	15	30	36	45	51	60	66	72	75	81	87	90	96	102	108	105	111	117	123	120	126	132	138	144
	5	10	20	25	30	35	40	45	50	50	55	60	60	65	70	75	70	75	80	85	80	85	90	95	100
	4	6	12	16	18	22	24	28	32	30	34	38	36	40	44	48	42	46	50	54	48	52	56	60	64
	3	3	6	9	9	12	12	15	18	15	18	21	18	21	24	27	21	24	27	30	24	27	30	33	36
	2	1	2	4	3	5	4	6	8	5	7	9	6	8	10	12	7	9	11	13	8	10	12	14	16
Y[1]	1	0	0	1	0	1	0	1	2	0	1	2	0	1	2	3	0	1	2	3	0	1	2	3	4
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	1	2	1	3	2	4	3	2	5	4	3	6	5	4	3	7	6	5	4	8	7	6	5	4
	-2	3	6	4	9	7	12	10	8	15	13	11	18	16	14	12	21	19	17	15	24	22	20	18	16
	-3	6	12	9	18	15	24	21	18	30	27	24	36	33	30	27	42	39	36	33	48	45	42	39	36
	-4	10	20	16	30	26	40	36	32	50	46	42	60	56	52	48	70	66	62	58	80	76	72	68	64
	-5	15	30	25	45	40	60	55	50	75	70	65	90	85	80	75	105	100	95	90	120	115	110	105	100
	-6	21	42	36	63	57	84	78	72	105	99	93	126	120	114	108	147	141	135	129	168	162	156	150	144
	-7	28	56	49	84	77	112	105	98	140	133	126	168	161	154	147	196	189	182	175	224	217	210	203	196
	-8	36	72	64	108	100	144	136	128	180	172	164	216	208	200	192	252	244	236	228	288	280	272	264	256
	-9	45	90	81	135	126	180	171	162	225	216	207	270	261	252	243	315	306	297	288	360	351	342	333	324
	-10	55	110	100	165	155	220	210	200	275	265	255	330	320	310	300	385	375	365	355	440	430	420	410	400

Figure 17. C003103 Atlas of Quadratic CGs in the Forms of Integer Number Sequences.

Four Sequence Variations (Figure 18): Expanding on the sequence analysis, this section showcases each sequence in four distinct forms: direct, reverse, negative direct, and negative reverse. Figure 18, accessible via [link], depicts these variations, enriching our understanding of sequence symmetries and transformations.

https://oeis.org/A000217				
C003110				
trio{1,0,0}, DES, |a|=0.5				
Tally	1	2	3	4
x_sp	-0.1	-0.1	0.1	0.1
y_sp	0.5	-0.5	0.5	-0.5
f	0	-1	0	-1
type	DES	DES	DES	DES
a	0.5	0.5	-0.5	-0.5
b	-0.5	0.5	0.5	-0.5
c	0	0	0	0
10	45	55	-45	-55
9	36	45	-36	-45
8	28	36	-28	-36
7	21	28	-21	-28
6	15	21	-15	-21
5	10	15	-10	-15
4	6	10	-6	-10
3	3	6	-3	-6
2	1	3	-1	-3
1	0	1	0	-1
0	0	0	0	0
-1	1	0	-1	0
-2	3	1	-3	-1
-3	6	3	-6	-3
-4	10	6	-10	-6
-5	15	10	-15	-10
-6	21	15	-21	-15
-7	28	21	-28	-21
-8	36	28	-36	-28
-9	45	36	-45	-36
-10	55	45	-55	-45

https://oeis.org/A002378				
C003120				
trio{2,0,0}, DES, |a|=1				
Tally	1	2	3	4
x_sp	-0.3	-0.3	0.3	0.3
y_sp	0.5	-0.5	0.5	-0.5
f	0	-1	0	-1
type	DES	DES	DES	DES
a	1	1	-1	-1
b	-1	1	1	-1
c	0	0	0	0
10	90	110	-90	-110
9	72	90	-72	-90
8	56	72	-56	-72
7	42	56	-42	-56
6	30	42	-30	-42
5	20	30	-20	-30
4	12	20	-12	-20
3	6	12	-6	-12
2	2	6	-2	-6
1	0	2	0	-2
0	0	0	0	0
-1	2	0	-2	0
-2	6	2	-6	-2
-3	12	6	-12	-6
-4	20	12	-20	-12
-5	30	20	-30	-20
-6	42	30	-42	-30
-7	56	42	-56	-42
-8	72	56	-72	-56
-9	90	72	-90	-72
-10	110	90	-110	-90

https://oeis.org/A000290				
C003130				
trio{1,0,1}, SUB, |a|=1				
Tally	1	2	3	4
x_sp	0	0	0	0
y_sp	0	0	0	0
f	0	0	0	0
type	SUB	SUB	SUB	SUB
a	1	1	-1	-1
b	0	0	0	0
c	0	0	0	0
10	100	100	-100	-100
9	81	81	-81	-81
8	64	64	-64	-64
7	49	49	-49	-49
6	36	36	-36	-36
5	25	25	-25	-25
4	16	16	-16	-16
3	9	9	-9	-9
2	4	4	-4	-4
1	1	1	-1	-1
0	0	0	0	0
-1	1	1	-1	-1
-2	4	4	-4	-4
-3	9	9	-9	-9
-4	16	16	-16	-16
-5	25	25	-25	-25
-6	36	36	-36	-36
-7	49	49	-49	-49
-8	64	64	-64	-64
-9	81	81	-81	-81
-10	100	100	-100	-100

https://oeis.org/A045943				
C003140				
trio{3,0,0}, DES, |a|=1.5				
Tally	1	2	3	4
x_sp	-0.4	-0.4	0.4	0.4
y_sp	0.5	-0.5	0.5	-0.5
f	0	-1	0	-1
type	DES	DES	DES	DES
a	1.5	1.5	-1.5	-1.5
b	-1.5	1.5	1.5	-1.5
c	0	0	0	0
10	135	165	-135	-165
9	108	135	-108	-135
8	84	108	-84	-108
7	63	84	-63	-84
6	45	63	-45	-63
5	30	45	-30	-45
4	18	30	-18	-30
3	9	18	-9	-18
2	3	9	-3	-9
1	0	3	0	-3
0	0	0	0	0
-1	3	0	-3	0
-2	9	3	-9	-3
-3	18	9	-18	-9
-4	30	18	-30	-18
-5	45	30	-45	-30
-6	63	45	-63	-45
-7	84	63	-84	-63
-8	108	84	-108	-84
-9	135	108	-135	-108
-10	165	135	-165	-135

https://oeis.org/A005449				
https://oeis.org/A000326				
C003150				
trio{2,0,1}, ACC, |a|=1.5				
Tally	1	2	3	4
x_sp	-0	-0	0	0
y_sp	0.2	-0.2	0.2	-0.2
f	0	0	0	0
type	ACC	ACC	ACC	ACC
a	1.5	1.5	-1.5	-1.5
b	-0.5	0.5	0.5	-0.5
c	0	0	0	0
10	145	155	-145	-155
9	117	126	-117	-126
8	92	100	-92	-100
7	70	77	-70	-77
6	51	57	-51	-57
5	35	40	-35	-40
4	22	26	-22	-26
3	12	15	-12	-15
2	5	7	-5	-7
1	1	2	-1	-2
0	0	0	0	0
-1	2	1	-2	-1
-2	7	5	-7	-5
-3	15	12	-15	-12
-4	26	22	-26	-22
-5	40	35	-40	-35
-6	57	51	-57	-51
-7	77	70	-77	-70
-8	100	92	-100	-92
-9	126	117	-126	-117
-10	155	145	-155	-145

Figure 18. C003104 Atlas of Quadratic CGs in the 4 Possible Forms of Integer Number Sequences: (1) Direct, (2) Reverse, (3) Negative Direct, (4) Negative Reverse.

10.2 Symmetry and Asymmetry in Quadratic CGs:

Symmetric Sequences: Sequences in the form of $trio = \{2a, 0{,}0\}$ are identified as DES-type symmetric, whereas those in the form of $trio = \{x_1, 0, x_1\}$ are classified as SUB-type symmetric.

Asymmetric Sequences: The $trio = \{x_1, 0, x_3\}$ configuration, for $x_1 \neq x_3$, represents asymmetric sequences, categorized as ACC-type.

10.3 Coefficient Analysis:

Integral Coefficients: Quadratic sequences with integral coefficients 'a' and 'b' can manifest as SUB, DES, and ACC types.

Non-Integral Coefficients: Sequences with non-integral coefficients exhibit only DES and ACC types.

Root Characterization: ACC-type sequences uniquely feature one integral and one non-integral root, whereas DES and SUB types possess exclusively integral roots. Furthermore, DES-type sequences are characterized by a y_{sp} (symmetry point) that is a multiple of $1/2$, while SUB-type sequences have an integral y_{sp}. ACC-type sequences, in contrast, have a non-integral, non-multiple of $1/2$ y_{sp}.

10.4 Method of Differences Analysis (Figure 19):

https://oeis.org/A000217

a= 0.5 | C003110 DES-type {1,0,0}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
1	9	32	33	34	35	36	37	38	39	40
1	8	24	25	26	27	28	29	30	31	32
1	7	17	18	19	20	21	22	23	24	25
1	6	11	12	13	14	15	16	17	18	19
1	5	6	7	8	9	10	11	12	13	14
1	4	2	3	4	5	6	7	8	9	10
1	3	-1	0	1	2	3	4	5	6	7
1	2	-3	-2	-1	0	1	2	3	4	5
1	1	-4	-3	-2	-1	0	1	2	3	4
1	0	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A002378

a= 1 | C003120 DES-type {2,0,0}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
2	18	68	69	70	71	72	73	74	75	76
2	16	52	53	54	55	56	57	58	59	60
2	14	38	39	40	41	42	43	44	45	46
2	12	26	27	28	29	30	31	32	33	34
2	10	16	17	18	19	20	21	22	23	24
2	8	8	9	10	11	12	13	14	15	16
2	6	2	3	4	5	6	7	8	9	10
2	4	-2	-1	0	1	2	3	4	5	6
2	2	-4	-3	-2	-1	0	1	2	3	4
2	0	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A000290

a= 1 | C003130 SUB-type {1,0,1}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
2	19	77	78	79	80	81	82	83	84	85
2	17	60	61	62	63	64	65	66	67	68
2	15	45	46	47	48	49	50	51	52	53
2	13	32	33	34	35	36	37	38	39	40
2	11	21	22	23	24	25	26	27	28	29
2	9	12	13	14	15	16	17	18	19	20
2	7	5	6	7	8	9	10	11	12	13
2	5	0	1	2	3	4	5	6	7	8
2	3	-3	-2	-1	0	1	2	3	4	5
2	1	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A045943

a= 1.5 | C003140 DES-type {3,0,0}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
3	27	104	105	106	107	108	109	110	111	112
3	24	80	81	82	83	84	85	86	87	88
3	21	59	60	61	62	63	64	65	66	67
3	18	41	42	43	44	45	46	47	48	49
3	15	26	27	28	29	30	31	32	33	34
3	12	14	15	16	17	18	19	20	21	22
3	9	5	6	7	8	9	10	11	12	13
3	6	-1	0	1	2	3	4	5	6	7
3	3	-4	-3	-2	-1	0	1	2	3	4
3	0	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A000326

a= 1.5 | C003150 ACC-type {2,0,1}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
3	28	113	114	115	116	117	118	119	120	121
3	25	88	89	90	91	92	93	94	95	96
3	22	66	67	68	69	70	71	72	73	74
3	19	47	48	49	50	51	52	53	54	55
3	16	31	32	33	34	35	36	37	38	39
3	13	18	19	20	21	22	23	24	25	26
3	10	8	9	10	11	12	13	14	15	16
3	7	1	2	3	4	5	6	7	8	9
3	4	-3	-2	-1	0	1	2	3	4	5
3	1	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A005449

a= 1.5 | C003150 ACC-type {1,0,2}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
3	29	122	123	124	125	126	127	128	129	130
3	26	96	97	98	99	100	101	102	103	104
3	23	73	74	75	76	77	78	79	80	81
3	20	53	54	55	56	57	58	59	60	61
3	17	36	37	38	39	40	41	42	43	44
3	14	22	23	24	25	26	27	28	29	30
3	11	11	12	13	14	15	16	17	18	19
3	8	3	4	5	6	7	8	9	10	11
3	5	-2	-1	0	1	2	3	4	5	6
3	2	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A046092

a= 2 | C003160 DES-type {4,0,0}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
4	36	140	141	142	143	144	145	146	147	148
4	32	108	109	110	111	112	113	114	115	116
4	28	80	81	82	83	84	85	86	87	88
4	24	56	57	58	59	60	61	62	63	64
4	20	36	37	38	39	40	41	42	43	44
4	16	20	21	22	23	24	25	26	27	28
4	12	8	9	10	11	12	13	14	15	16
4	8	0	1	2	3	4	5	6	7	8
4	4	-4	-3	-2	-1	0	1	2	3	4
4	0	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A000384

a= 2 | C003170 ACC-type {3,0,1}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
4	37	149	150	151	152	153	154	155	156	157
4	33	116	117	118	119	120	121	122	123	124
4	29	87	88	89	90	91	92	93	94	95
4	25	62	63	64	65	66	67	68	69	70
4	21	41	42	43	44	45	46	47	48	49
4	17	24	25	26	27	28	29	30	31	32
4	13	11	12	13	14	15	16	17	18	19
4	9	2	3	4	5	6	7	8	9	10
4	5	-3	-2	-1	0	1	2	3	4	5
4	1	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A001105

a= 2 | C003180 SUB-type {2,0,2}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
4	38	158	159	160	161	162	163	164	165	166
4	34	124	125	126	127	128	129	130	131	132
4	30	94	95	96	97	98	99	100	101	102
4	26	68	69	70	71	72	73	74	75	76
4	22	46	47	48	49	50	51	52	53	54
4	18	28	29	30	31	32	33	34	35	36
4	14	14	15	16	17	18	19	20	21	22
4	10	4	5	6	7	8	9	10	11	12
4	6	-2	-1	0	1	2	3	4	5	6
4	2	-4	-3	-2	-1	0	1	2	3	4

https://oeis.org/A014105

a= 2 | C003170 ACC-type {1,0,3}

dif_2	dif_1	-4	-3	-2	-1	0	1	2	3	4
4	39	167	168	169	170	171	172	173	174	175
4	35	132	133	134	135	136	137	138	139	140
4	31	101	102	103	104	105	106	107	108	109
4	27	74	75	76	77	78	79	80	81	82
4	23	51	52	53	54	55	56	57	58	59
4	19	32	33	34	35	36	37	38	39	40
4	15	17	18	19	20	21	22	23	24	25
4	11	6	7	8	9	10	11	12	13	14
4	7	-1	0	1	2	3	4	5	6	7
4	3	-4	-3	-2	-1	0	1	2	3	4

Figure 19. C003105 Atlas of Quadratic CGs ((matrix of tables obtained by the reverse path of the finite difference method).

Employing the inverse approach of the method of finite differences, this section elucidates the underlying structure of these sequences. The last row of each table aligns with the integer number sequence, centering on zero. A notable observation is the 45-degree counterclockwise diagonal rotation across the horizontal sequence of tables, offering insights into the dynamic behavior of zeros in quadratic CGs. This phenomenon also reveals the impact of increasing the coefficient 'a' on the distribution of zeros around the central zero, which is instrumental in understanding the limits of prime number sequences within these CGs.

10.5 Fundamental Table of Quadratic CGs (Figure 20):

SNYPO	Trio	Type	a	OEIS	OEIS Reverse
C003110	{1,0,0}	DES	0.5	https://oeis.org/A000217	
C003120	{2,0,0}	DES	1	https://oeis.org/A002378	
C003130	{1,0,1}	SUB	1	https://oeis.org/A000290	
C003140	{3,0,0}	DES	1.5	https://oeis.org/A045943	
C003150	{2,0,1}	ACC	1.5	https://oeis.org/A000326	https://oeis.org/A005449
C003160	{4,0,0}	DES	2	https://oeis.org/A046092	
C003170	{3,0,1}	ACC	2	https://oeis.org/A000384	https://oeis.org/A014105
C003180	{2,0,2}	SUB	2	https://oeis.org/A001105	
C003190	{5,0,0}	DES	3.5	https://oeis.org/A028895	
C003200	{4,0,1}	ACC	3.5	https://oeis.org/A000566	https://oeis.org/A147875
C003210	{3,0,2}	ACC	3.5	https://oeis.org/A005476	https://oeis.org/A005475
C003220	{6,0,0}	DES	3	https://oeis.org/A028896	
C003230	{5,0,1}	ACC	3	https://oeis.org/A000567	https://oeis.org/A045944
C003240	{4,0,2}	ACC	3	https://oeis.org/A049450	https://oeis.org/A049451
C003250	{3,0,3}	SUB	3	https://oeis.org/A033428	
C003260	{7,0,0}	DES	3.5	https://oeis.org/A024966	
C003270	{6,0,1}	ACC	3.5	https://oeis.org/A001106	https://oeis.org/A179986
C003280	{5,0,2}	ACC	3.5	https://oeis.org/A218471	https://oeis.org/A186029
C003290	{4,0,3}	ACC	3.5	https://oeis.org/A022264	https://oeis.org/A022265
C003300	{8,0,0}	DES	4	https://oeis.org/A033996	
C003310	{7,0,1}	ACC	4	https://oeis.org/A001107	https://oeis.org/A033954
C003320	{6,0,2}	ACC	4	https://oeis.org/A002939	https://oeis.org/A002943
C003330	{5,0,3}	ACC	4	https://oeis.org/A033991	https://oeis.org/A007742
C003340	{4,0,4}	SUB	4	https://oeis.org/A016742	
C003350	{9,0,0}	DES	4.5		
C003360	{8,0,1}	ACC	4.5		
C003370	{7,0,2}	ACC	4.5		
C003380	{6,0,3}	ACC	4.5		
C003390	{5,0,4}	ACC	4.5		
C003400	{10,0,0}	DES	5		
C003410	{9,0,1}	ACC	5		
C003420	{8,0,2}	ACC	5		
C003430	{7,0,3}	ACC	5		
C003440	{6,0,4}	ACC	5		
C003450	{5,0,5}	SUB	5		

Figure 20. C003109 Atlas of Quadratic CGs (Summary Table).

This summary table classifies all trio forms and their corresponding types, linking each to relevant sequences in the OEIS and SNYPO for deeper exploration. For instance, the trio {1,0,0} is categorized as DES-type with a coefficient of 0.5, linked to its OEIS sequence https://oeis.org/A000217. This table serves as a quick reference guide for researchers, facilitating easy access to comprehensive data on quadratic CGs.

11 Reflections on the Duplicity of Asymmetric Polynomial Sequences in OEIS

The OEIS adopts a distinctive approach to cataloging asymmetric polynomial sequences, characterized by creating dual identities. This approach, evident in all notes and references within the OEIS, suggests that each asymmetric sequence is bifurcated into two complementary sequences. Such practice mirrors the inherent symmetry in the curves and polynomial formulas of the sequences.

Analyzing the sequences https://oeis.org/A049450 and https://oeis.org/A049451, we observe Philippe Deléham's work illustrating the growth of the sequences for indices $n \geq 0$. The geometric configuration of square numbers added to the products $n(2n-1)$ (integers multiplied by odd numbers) provides valuable insights into the structure of these sequences:

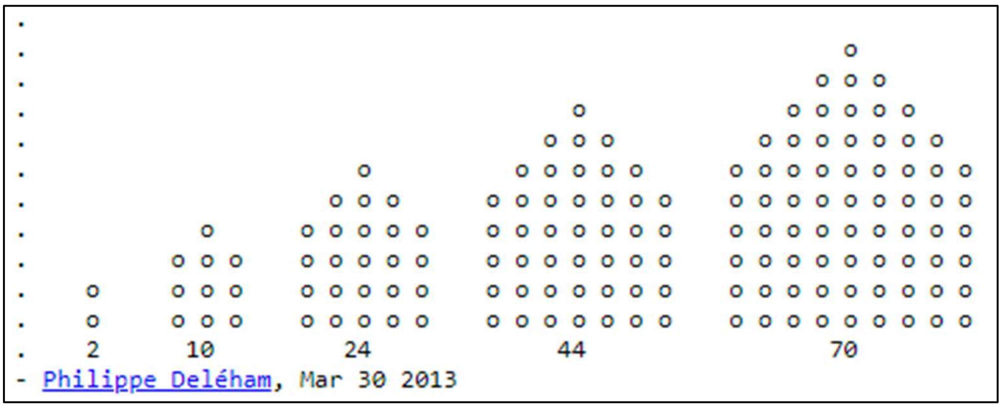


Figure 21. The growth of $a(n) = n(3n-1)$ from https://oeis.org/A049450.

The applicability of the formulas for these sequences to both positive and negative indices is a key point, highlighting the universality of their properties. The expression $a(n) = n(3n-1) = 3n^2 - n$, for example, can be rewritten as $a(n) = n^2 + n(2n-1)$, underscoring the presence of the zero element in the sequence.

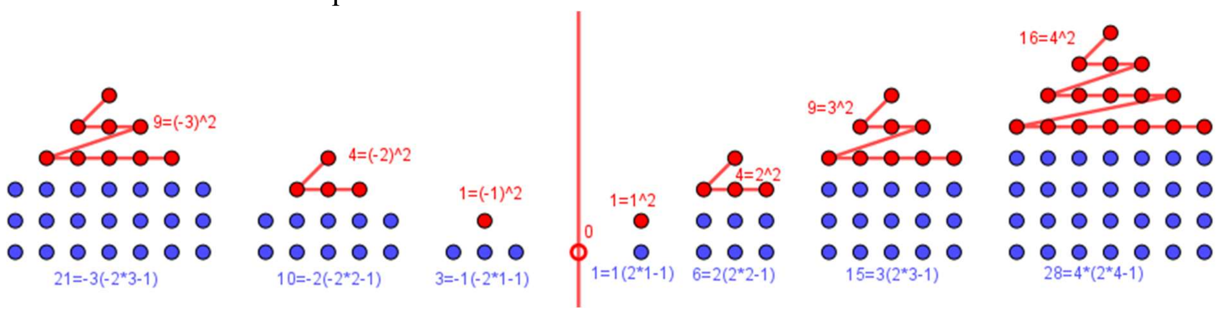


Figure 22. $a(n) = n(3n-1) = C003250 = /A049451/ + A049450.ggb.$

These observations are grounded in the method of finite differences, applicable to all elements of a polynomial sequence, regardless of the offset. The computational properties, including integrals and derivatives, are valid across the entire span of the sequence.

The classification of polynomial sequences deserves special attention. For example, the sequence $trio = \{2a, 0,0\}$ implies a form $quartet = \{2a, 0,0.2a\}$. Thus, we classify the asymmetric sequence $\{x_1, 0, x_3\}$ as identical to $\{x_3, 0, x_1\}$ in reverse, and the same applies to their negative forms. In our approach, we consider the following variations of trios as a single quadratic sequence:

$$\{x_1, 0, x_3\} == \{x_3, 0, x_1\} == \{-x_1, 0, -x_3\} == \{-x_3, 0, -x_1\}$$

This perspective, though it has met resistance in some circles within the OEIS, seeks to deepen our understanding of polynomial sequences and their intricate properties.

12 Quadratic CG Sequences Matrix Universe

To systematically organize the realm of quadratic CG sequences, we are set to create a universe of matrices dedicated to these CGs. This universe of matrices is designed to methodically arrange and showcase the attributes of quadratic CG sequences. The significance of organizing the Quadratic CG Sequences Matrix Universe is akin to the importance that demonstration figures held in the proofs of the 3 basic quadratic theorems. Hence, these matrices play an integral role in attempting to solve a variety of longstanding, yet unresolved, mathematical problems, including potentially providing solutions for Goldbach's Conjecture, Landau's Problems, and others.

Each matrix is based on the "quadratic CG sequence". Every quadratic CG conforms to the form $Y2[y] = \{x_1, 0, x_3\}$. The inclusion of at least one zero element is a defining characteristic of these quadratic CGs, leading to the term "CG" being an acronym for Composite Generator. The Composite Generator concept relates to the sequence's inherent capacity to yield either none or a finite number of prime numbers while producing an infinite sequence of composite numbers.

12.1 Eureka Matrix

Each matrix or table in question features a central quadratic CG sequence in the form of $Y2[y] = \{x_1, 0, x_3\}$.

For the columns to the right of the central quadratic CG, one unit is added to each element of the sequence, and this process is progressively repeated for each subsequent column to the right. Similarly, for the columns to the left, one unit is subtracted from each element of the sequence, and this operation continues for each subsequent column to the left. The result of this construction is a table that displays progressive variations of the original quadratic CG sequence, demonstrating an arithmetic increase and decrease of the elements as we move to the right and left, respectively, from the central column.

The name "Eureka" was inspired by the links "Eureka Sequences" at https://www.curioustem.org/stem-articles/eureka-sequences and "Eureka Sequences - Numberphile" at https://www.youtube.com/watch?v=6X2D497is6Y.

In all Eureka matrices, there is a central zero. Thus, its vertical and the diagonals at ±45º are always quadratic CGs.

12.2 $Y2[y] - Y2[f]$ Sequences Matrix

In our exploration of the vast landscape of quadratic CGs, each uniquely defined by the trio $\{x_1, 0, x_3\}$, we embark on a systematic construction of an endless array of matrices. These matrices are designated as '$Y2[y] - Y2[f]$ sequences matrices.' Central to this construction is $Y2[y] = \{x_1, 0, x_3\}$, which serves as the foundational quadratic CG for each matrix.

The process hinges on the subtraction operation $Y2[y] - Y2[f]$. This operation is pivotal, as it ensures that each quadratic CG, $Y2[y] = \{x_1, 0, x_3\}$, invariably incorporates the zero element. This consistent inclusion of zero is not merely a repetitive pattern but a defining characteristic that shapes the nature of each matrix.

Consequently, each quadratic CG, through the mechanism of $Y2[y] - Y2[f]$, gives rise to its unique '$Y2[y] - Y2[f]$ sequences matrix.' This matrix is not just a simple collection of numbers; it is a rich tapestry of infinite sequences, each a CG in its own right, perpetually echoing the zero element from the original trio $\{x_1, 0, x_3\}$.

12.3 Variant Multiplication Table (VMT)

In the boundless universe of quadratic CGs, an infinity of multiplication tables is produced, each exhibiting unique variations through rotations or offsets. We designate each distinct multiplication table within this array as a 'Variant Multiplication Table' (VMT). This term captures their individualized nature, differing from the standard multiplication model through specific alterations.

13 Universe of Quadratic CG Sequences Matrices on Mersenne Forum

To initiate our universe of matrices, let us start with the trio of zeros {0,0,0}. The first table will be C003106 $trio\{0,0,0\}$, $a = 0$, Eureka Matrix 20x20.

Col.	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
x_sp	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####
y_sp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
type	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB
a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
c	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
19	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
18	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
17	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
16	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
15	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
14	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
13	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
12	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
10	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
9	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
8	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
7	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
6	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-2	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-3	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-4	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-5	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-6	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-7	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-8	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-9	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-10	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-11	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-12	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-13	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-14	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-15	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-16	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-17	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-18	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-19	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-20	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 23. C003106 $trio\{0,0,0\}$, $a = 0$, Eureka Matrix 20x20.

Note that for each Eureka Matrix, the same universe of constants will run in parallel. When these are folded according to the parabolic curves of each trio, they will form the distribution of all composite and prime constants across the XY plane.

This property enables us to demonstrate that all quadratic sequences (and later we will show this for any polynomial sequence) of integers that are not CGs nor reducible, contain a countless number of prime numbers. Furthermore, it becomes evident that no quadratic sequence of prime numbers can exceed the length of Euler's famous sequence, which is the oblong sequence plus 41, represented as $\{oblong + 41\} = \{2,0,0\} + 41 = \{43,41,41\}$. This highlights a fundamental limit in the size of prime number sequences within the quadratic framework.

The second table will be C003107 $trio\{0,0,0\}$, $a = 0$, $Y2(y)$–$Y2(f)$ Sequences Matrix 15x15.

PS[0 x^2+(0)x+(0), 0 x^2+(0)x+(0), 0 x^2+(0)x+(0)]

							y_sp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							type	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB
							a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
y_sp	f	type	a	b	c		X-axis -->	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	SUB	0	0	0	Y-axis -->	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0	Y[1]	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0	Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0	Y[-1]	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	0	0		-15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Figure 24. C003107 $trio\{0,0,0\}$, $a = 0$, $Y2(y)–Y2(f)$ Sequences Matrix 15x15.

The third table will be C003108 $trio\{0,0,0\}$, $a = 0$, Variant MT (VMT) 15x15.

															PS[(	0	)x^2+(	1	)x+(	0	),	0	x^2+(	0	)x+(	0	),	0	x^2+(	-1	)x+(	0	)]					
							y_sp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							type	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB	SUB
							a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							b	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
							c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
y_sp	f	type	a	b	c		X-axis -->	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	SUB	0	-15	0	Y-axis -->	15	225	210	195	180	165	150	135	120	105	90	75	60	45	30	15	0	-15	-30	-45	-60	-75	-90	-105	-120	-135	-150	-165	-180	-195	-210	-225
0	0	SUB	0	-14	0		14	210	196	182	168	154	140	126	112	98	84	70	56	42	28	14	0	-14	-28	-42	-56	-70	-84	-98	-112	-126	-140	-154	-168	-182	-196	-210
0	0	SUB	0	-13	0		13	195	182	169	156	143	130	117	104	91	78	65	52	39	26	13	0	-13	-26	-39	-52	-65	-78	-91	-104	-117	-130	-143	-156	-169	-182	-195
0	0	SUB	0	-12	0		12	180	168	156	144	132	120	108	96	84	72	60	48	36	24	12	0	-12	-24	-36	-48	-60	-72	-84	-96	-108	-120	-132	-144	-156	-168	-180
0	0	SUB	0	-11	0		11	165	154	143	132	121	110	99	88	77	66	55	44	33	22	11	0	-11	-22	-33	-44	-55	-66	-77	-88	-99	-110	-121	-132	-143	-154	-165
0	0	SUB	0	-10	0		10	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100	-110	-120	-130	-140	-150
0	0	SUB	0	-9	0		9	135	126	117	108	99	90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90	-99	-108	-117	-126	-135
0	0	SUB	0	-8	0		8	120	112	104	96	88	80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72	-80	-88	-96	-104	-112	-120
0	0	SUB	0	-7	0		7	105	98	91	84	77	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63	-70	-77	-84	-91	-98	-105
0	0	SUB	0	-6	0		6	90	84	78	72	66	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54	-60	-66	-72	-78	-84	-90
0	0	SUB	0	-5	0		5	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75
0	0	SUB	0	-4	0		4	60	56	52	48	44	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40	-44	-48	-52	-56	-60
0	0	SUB	0	-3	0		3	45	42	39	36	33	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30	-33	-36	-39	-42	-45
0	0	SUB	0	-2	0		2	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20	-22	-24	-26	-28	-30
0	0	SUB	0	-1	0	Y[1]	1	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
0	0	SUB	0	0	0	Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	SUB	0	1	0	Y[-1]	-1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	SUB	0	2	0		-2	-30	-28	-26	-24	-22	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
0	0	SUB	0	3	0		-3	-45	-42	-39	-36	-33	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
0	0	SUB	0	4	0		-4	-60	-56	-52	-48	-44	-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
0	0	SUB	0	5	0		-5	-75	-70	-65	-60	-55	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
0	0	SUB	0	6	0		-6	-90	-84	-78	-72	-66	-60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
0	0	SUB	0	7	0		-7	-105	-98	-91	-84	-77	-70	-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
0	0	SUB	0	8	0		-8	-120	-112	-104	-96	-88	-80	-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
0	0	SUB	0	9	0		-9	-135	-126	-117	-108	-99	-90	-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
0	0	SUB	0	10	0		-10	-150	-140	-130	-120	-110	-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
0	0	SUB	0	11	0		-11	-165	-154	-143	-132	-121	-110	-99	-88	-77	-66	-55	-44	-33	-22	-11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
0	0	SUB	0	12	0		-12	-180	-168	-156	-144	-132	-120	-108	-96	-84	-72	-60	-48	-36	-24	-12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
0	0	SUB	0	13	0		-13	-195	-182	-169	-156	-143	-130	-117	-104	-91	-78	-65	-52	-39	-26	-13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
0	0	SUB	0	14	0		-14	-210	-196	-182	-168	-154	-140	-126	-112	-98	-84	-70	-56	-42	-28	-14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
0	0	SUB	0	15	0		-15	-225	-210	-195	-180	-165	-150	-135	-120	-105	-90	-75	-60	-45	-30	-15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Figure 25. C003108 $trio\{0,0,0\}$, $a = 0$, Variant MT (VMT) 15x15.

This last table is precisely the well-known Multiplication Table, as studied in "Kusniec, Charles. (2023). The Hyperbolic Sieve of Prime Numbers. Zenodo. https://doi.org/10.5281/zenodo.8419575. [19]

Currently, the continuation of this work will be presented and discussed in the Mersenne Forum as follows.

[B]Announcement of Publication: Universe of Quadratic CG Sequences Matrices on Mersenne Forum[/B]

Dear Mersenne Forum Members,

I am excited to share with you a project that has been a passion of mine for the past five years: the [URL="https://www.mersenneforum.org/showthread.php?t=28116"]"Universe of Quadratic CG Sequences Matrices."[/URL] This endeavor represents a unique approach to understanding quadratic sequences and their mathematical applications.

Through this project, I have developed a series of matrices, which I believe are crucial tools for addressing complex mathematical problems. However, I want to be transparent about certain aspects of this publication:

[B]1. Limitations of the Current Medium:[/B] I acknowledge that a blog may not be the ideal platform for presenting these matrices. The lack of versatility compared to a dynamic medium like MS-Excel is a constraint. In Excel, changing a single value can alter all related matrices, a feature not easily replicated on a blog. Nevertheless, this platform is the best available option for me at this time.

[B]2. Scope of the Tables:[/B] My initial ambition was to generate tables covering all variations up to a=5, which would include a=4—a significant square value—and allow for a comprehensive comparison of the sequences before and after this point. This endeavor would result in at least 105 tables except 10 introductory tables. However, due to the immense scope of this task, I decided to limit the project to a=2, resulting in a total of 24+10 tables. This decision was made to maintain manageability and focus on the project.

For those interested in the mathematical foundation behind this work, I am pleased to inform you that all the detailed mathematical underpinnings can be found in the preprint "The SOM Numbers (part I)," last version available at [url]https://doi.org/10.5281/zenodo.10208029[/url].

Additionally, if anyone is interested in proposing to automate the generation of these matrices, it would be highly beneficial. Ideally, such a program would automatically link each sequence generated to its corresponding entry in the OEIS, especially for each horizontal, vertical, and ±45-degree diagonals of each matrix. This would greatly enhance the utility and interactivity of the matrices, providing direct access to a wealth of related information and facilitating further research.

In light of this, I would like to request the Mersenne Forum administrators to consider upgrading my user status to enable me to edit my posts, particularly in threads related to this matrix universe. Furthermore, if it aligns with the forum's policies and capabilities, perhaps a dedicated blog or space could be opened for exclusively hosting these tables. This option could provide a more suitable platform for this kind of content, allowing for greater flexibility and engagement from the community. This ability to edit, and potentially to host this content in a dedicated space, is vital for ensuring accuracy and incorporating new insights as my research progresses.

I fully respect the forum's guidelines and assure you that any edits or contributions I make will adhere to these standards. Your consideration is greatly appreciated.

Thank you for your support, and I look forward to your engagement with this project. I believe that the [B]"Universe of Quadratic CG Sequences Matrices"[/B] will not only deepen our understanding of quadratic sequences but also open new avenues for mathematical exploration.

Sincerely,

Charles Kusniec.

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I extend my deepest gratitude to Mr. H. Bli Shem and my Family for their unwavering support and constant inspiration throughout this journey. As a Talmud proverb profoundly states, “*He who does not teach his son a trade teaches him to steal.*” Among the wisest of trades is the pursuit of knowledge through study. I dedicate these studies to my beloved children.

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